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MANIPULATIVE MATERIALS IN THE TEACHING
OF PROBLEM SOLVING

by



ROBERT MATHIAS SCHERER

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Manipulative Materials in the Teaching of Problem Solving," submitted by Robert Mathias Scherer in partial fulfillment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of this study was to determine the effectiveness of using manipulative materials to develop problem solving ability.

Eight classes of grade three pupils from the Edmonton Separate School System were selected. Four of the classes were assigned to the control group and the other four classes formed the experimental group.

After pretesting the pupils to estimate their problem solving ability, both groups were given a treatment consisting of eight lessons designed to develop ability to solve verbal arithmetic problems. The treatment for each group was the same except that the experimental group used a variety of manipulative materials in their discussion of verbal problems. No manipulative materials were used by the control group.

Following the treatment, the problem solving ability of the subjects was tested again. The amount of improvement in ability to write correct equations and to arrive at correct numerical answers that could be attributed to each of the treatment methods was compared. The comparisons were analyzed to determine if the use of manipulative materials resulted in significantly more ability to solve additive and subtractive problems.

One month after the treatment the test was administered a third time. Statistical analyses were conducted to determine if the use of manipulative materials had a significant effect on retention of ability to write correct equations or to arrive at correct numerical answers for

additive and subtractive problems.

To determine if the use of manipulative materials was more effective for any specific sector of the school population, separate comparisons were made between the control and experimental groups for each of the following subgroups: boys, girls, pupils of high intelligence, pupils of average intelligence, pupils of low intelligence, pupils of high socio-economic status and pupils of low socio-economic status. Comparisons were made for each subgroup on both the post test and retention test scores.

The findings of the study led to the conclusion that a method of instruction which emphasizes the use of manipulative materials for a short period of time is no more effective in developing ability to solve additive and subtractive problems than is a method using only discussion and printed materials.

There was some indication that the use of manipulative materials may aid boys in their retention of ability to arrive at correct numerical answers for additive and subtractive problems.

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CHAPTER I

THE PROBLEM, ITS NATURE AND SIGNIFICANCE

I. INTRODUCTION

Through the years theories of learning have continuously influenced the methods and materials used to teach arithmetic in the elementary school. Since the 1920's two theories have had notable influence upon the teaching of arithmetic.

During the third and fourth decades of this century, the influence of Thorndike's "connection" or "stimulus-response" theory had a marked effect upon educational practice. Arithmetic textbooks and methods of instruction were evaluated in light of this theory. Drill type exercises were stressed to develop skill in making speedy responses. An attempt was made to maintain the attention of the students through the use of games, races and other types of motivation.

With the development of the "field theory" of learning or Gestalt Psychology during the 1940's a shift began to take place in the teaching of arithmetic. Concept formation through understanding began to replace the stress on learning by rote memorization. It became important that the pupil understand the structure of the concepts to be learned rather than merely memorize them.

With the stress on understanding came a renewed emphasis on the use of concrete instructional materials. According to Buswell (1951, p. 152) the field theorists insisted on materials that would lead the

pupil from a concrete portrayal or understanding of a process to an abstract representation in the form of mathematical symbols.

One area of the arithmetic program where the field theories have had a great influence is in the teaching of verbal problem solving. As Lindstedt (1960, p. 21) pointed out, these theories hold that all problems begin as situations and must be structured before the solution can be meaningful to the pupil. The equation approach to problem solving has thus become popular because it supplies a method of structuring the problem and, it is claimed, makes problem solving mathematically meaningful.

The use of various kinds of instructional materials has been recommended as a means of making the child aware of the structure of problems. It was thought that this would improve the pupil's ability to solve arithmetic problems. Sudduth (1962, p. 40) in her survey of educational journals reported forty-five different recommendations for using instructional materials to teach problem solving. In her review, however, she reported no studies done to investigate the effectiveness of the use of these materials. She recommended more studies be done to determine the effectiveness of various materials in the teaching of arithmetic (Sudduth, 1962, p. 57).

Grossnickle (quoted in Buswell, 1951, p. 285), in proposing a problem for research in the teaching and learning of arithmetic, also suggested that a study be undertaken to determine the effectiveness of the use of manipulative materials in the teaching of arithmetic.

It is the purpose of this study to investigate the effectiveness

of the use of manipulative materials in improving the ability of grade three pupils to solve additive and subtractive verbal arithmetic problems.

II. THE HYPOTHESES

Instruction in most modern elementary mathematics programs is based on the premise that problem solving is meaningful only when the structure of the problem is understood by the pupil. Research appears to be necessary, however, to determine what instructional techniques and materials will best foster the thinking necessary for this understanding.

Basically, this study was attempted in an effort to answer the following two questions:

1. Do pupils who use manipulative materials during problem solving instruction develop significantly more ability to solve verbal arithmetic problems than do pupils who use only a discussion method of instruction?

2. Do pupils who use manipulative materials during problem solving instruction retain significantly more ability to solve verbal arithmetic problems than do pupils who use only a discussion method of instruction?

To answer these questions four null hypotheses were tested. An attempt was made to determine if the use of manipulative materials was more useful to any subgroup of the pupils exposed to this type of instruction. For this reason each null hypothesis was tested for each

of the following subgroups of subjects: all subjects, all girls, all boys, all subjects of high intelligence, all subjects of low intelligence, all subjects of low socio-economic status, and all subjects of high socio-economic status.

The null hypotheses were:

Hypothesis 1. Pupils using manipulative materials during problem solving instruction do not develop significantly more ability to write equations representing the structure of additive and subtractive arithmetic problems than do pupils who use only a discussion method of instruction, when initial differences in reading comprehension and computational skills are controlled.

Hypothesis 2. Pupils using manipulative materials during problem solving instruction do not develop significantly more ability to arrive at correct numerical answers for additive and subtractive problems than do pupils who use a discussion method of instruction, when initial differences in reading comprehension and computational skills are controlled.

Hypothesis 3. Pupils using manipulative materials during problem solving instruction do not retain significantly more ability to write equations representing the structure of additive and subtractive problems than do pupils who use only a discussion method of instruction.

Hypothesis 4. Pupils using manipulative materials during problem solving instruction do not retain significantly more ability to arrive at

correct numerical answers for additive and subtractive problems than do pupils who use only a discussion method of instruction.

III. BASIC ASSUMPTIONS

This investigation was based on the following assumptions:

1. It was assumed that the materials which were designed for this study and the methods by which they were used are conducive to the development of verbal problem solving ability.

2. It was assumed that the tests employed in the study were adequate measures of the attributes they were designed to measure.

3. It was assumed that the participating teachers were comparable in capability and motivation to elicit growth in problem solving ability.

4. The assumption was made that a period of eight lessons was sufficient to yield any differences in problem solving ability that may result from the experimental method.

5. Eight classrooms, four experimental and four control, were assumed to be of sufficient number to give validity to any conclusions which might result from the study.

6. It was assumed that the eight classes of pupils used in this study were representative of the total population of grade three students in the school system.

IV. LIMITATIONS

This study had the following limitations:

1. The study was limited to those students who used the grade

three textbook of the Seeing Through Arithmetic (1959 edition) series.

2. Only additive and subtractive problems were considered in this study; thus, generalizations cannot be made to all types of verbal arithmetic problems.

3. The study was limited to the extent that there may have been an interaction between the experimental method and the previous experiences of the experimental group which was not accounted for by the statistical procedures used.

4. The study was limited to the extent that the use of the same problem solving test three times during the experiment may have affected the pupils' scores.

V. DEFINITION OF TERMS

Verbal Arithmetic Problem. A quantitative situation stated in words so that the essential structuring and selection of the computational process necessary for its solution must be determined by the problem solver.

Verbal Problem Solving. In this study, verbal problem solving will be defined as having the following steps: (1) reading and comprehending the problem and the question it asks, (2) mentally structuring the problem to determine the sequence of events suggested by the problem, (3) writing an equation using symbols to represent the structure of the problem, (4) computing to determine the solution to the equation, and (5) stating the solution in verbal form.

Structure. The internal order or 'action' of the problem which

if understood results in insight and meaningful interpretation.

Additive Problems. Those problems in which the situation described is one of combining objects either in the physical sense or mentally by imagination.

Subtractive Problems. Those problems in which the situation described is one of separating a subgroup of objects from a group either in the physical sense or mentally by imagination.

Manipulative Materials. Those concrete materials that can be used by the student to demonstrate the additive or subtractive action suggested by the situation described by the problem.

Subjects of High Intelligence (High I.Q.). Those pupils whose I.Q. scores on the Otis Quick-Scoring Mental Ability Test, Alpha Form A were 120 or greater.

Subjects of Average Intelligence (Average I.Q.). Those pupils whose I.Q. scores on the Otis Quick-Scoring Mental Ability Test, Alpha Form A were less than 120 but greater than 104.

Subjects of Low Intelligence (Low I.Q.). Those pupils whose I.Q. scores on the Otis Quick-Scoring Mental Ability Test, Alpha Form A were 104 or less.

Subjects of High Socio-economic Status (High S.E.S.). Those pupils whose guardian had a socio-economic rating of 52.0 or greater on the Blishen Occupational Class Scale.

Subjects of Low Socio-economic Status (Low S.E.S.). Those pupils whose guardian had a socio-economic rating of less than 52.0 on the Blishen Occupational Class Scale.

VI. THE EXPERIMENTAL SETTING

The experimental design and statistical procedures used for analysis are fully described in Chapter III. A brief summary of the setting is given here as an introduction.

The population from which the sample was drawn consisted of the total grade three enrollments of six elementary schools of the Edmonton Separate School System. These six schools contained ten classes of pupils. Two of the schools were used in the pilot study. The remaining four schools, containing eight classes, were involved in the main part of the experiment. The eight classes participating in the experiment were matched in pairs on the following basis:

1. Each class contained pupils of similar ability as described by the classroom teacher.
2. The size of each class was approximately the same.
3. Each class had similar arithmetic experiences at school previous to the experiment. All classes involved in the experiment used the grade three textbook of Seeing Through Arithmetic (1959) series.
4. Each class spent about the same amount of time on arithmetic instruction during the school day.
5. The extent concrete materials were used was approximately the same for each class previous to the study.

The classes were assigned to the control group and the experimental group so that one class from each school was in each group.

At the beginning of the experiment, each subject was given a test designed to estimate his ability to solve additive and subtractive

problems. Both the experimental group and the control group were then given a treatment consisting of a series of eight lessons designed to teach verbal arithmetic problem solving. The treatment was designed to emphasize the use of manipulative materials by the subjects in the experimental group. In the treatment for the control group no manipulative materials were used and a discussion method of instruction was stressed.

Immediately after the treatment, the subjects were again tested to determine the amount of gain in ability to solve additive and subtractive problems that could be attributed to the methods used in the treatment. The same test was administered again approximately one month after the treatment. The second post test was to ascertain whether the use of manipulative materials by the experimental group resulted in greater retention of the ability to solve verbal arithmetic problems.

VII. THE OUTLINE OF THE REPORT

In Chapter I of this report the problem is introduced. Chapter II contains a detailed review of the related literature. Chapter III contains a detailed description of each aspect of the experimental design, the methods and materials used in the study, and the statistical analysis used to test the hypotheses. The data analysis of the study is contained in Chapter IV. Finally, Chapter V includes a summary of the study, reports the conclusions and implications, and presents some suggestions for further research.

CHAPTER II

REVIEW OF RELATED RESEARCH AND LITERATURE

There has been a great variety of research on the subject of problem solving in the past half century. In that time the development of logical thought in children has also received considerable attention. It is only recently, however, that an effort has been made to relate ability in problem solving in children to the development of logical thought. During this period there has been a steady increase in the use of concrete materials as an aid to thinking in young children.

Literature related to this study is considered in this chapter. The first section of this chapter is concerned with what constitutes a verbal arithmetic problem. In the second part of this chapter consideration is given to how children think and how childish thought can be related to the thinking required to solve arithmetic problems. In the third portion of the chapter the factors that constitute problem solving ability are discussed and methods for improving problem solving ability are suggested. The fourth part of the chapter contains a discussion of the trend toward the use of manipulative materials in teaching arithmetic. Reference also is made to research concerning the effectiveness of using manipulative materials to teach verbal arithmetic problem solving. The final portion of the chapter is devoted to a summary of the factors which constitute problem solving ability and of methods to improve this ability.

I. WHAT CONSTITUTES A VERBAL PROBLEM?

The debate of what constitutes a verbal arithmetic problem has continued for several decades now. As early as 1930 Morton (1937, p. 107) defined a problem as an arithmetical situation stated in words which required the pupil to first decide which operation to perform. He classified only those problems which arose out of experiences in which the pupils engaged in actual concrete situations as true arithmetic problems. Thus, statements which appeared in books were not considered true problems since they did not arise out of the experiences of the children who solved them. He believed, however, that rich sources of printed problem material should be used by the teacher to supplement the material which came out of activities in which pupils engaged. Such supplementary problems were needed because of the probability that an insufficient number would come from the children's activities. Also, supplementary problems could provide a better balance and a better distribution of practice.

At about the same time as Morton gave his views, Wheat expounded his own opinions on what constituted the proper emphasis of the verbal arithmetic problem. In discussing the new trends of the time he wrote:

The 'old-fashioned' method shows the pupil how to perform the operations of arithmetic: that is, teaches the 'number facts' in both simple and complex operations, and finally shows the pupils how to 'solve problems'. The 'modern' method presents quantitative situations as 'problems' to be solved, and by devious ways leads from the 'problems' to the particular operations used in the process of 'solving' (Wheat, 1937, p. 134).

To Wheat, neither of these methods were really dealing with

problems since true problems arose only out of doubt about the way in which the situation should be handled. It was only when there was ignorance, or complete uncertainty, about the method with which the conditions should be dealt that the quantitative situation had the character of a problem. The 'problem' in arithmetic was in reality not a problem at all. Rather, it was a type of illustrative exercise where the pupil was asked to solve some quantitative situation according to a prearranged method.

More recently, the concept of a verbal arithmetic problem has been defined by a statement of the conditions that are necessary for the process of problem solving to take place. Henderson and Pingry listed three such conditions:

1. The individual has a clearly defined goal of which he is consciously aware and whose attainment he desires.
2. Blocking of the path toward the goal occurs, and the individual's fixed patterns of behavior or habitual responses are not sufficient for removing the block.
3. Deliberation takes place. The individual becomes aware of the problem, defines it more or less clearly, identifies various possible solutions, and tests these for feasibility (Henderson and Pingry, 1961, p. 230).

According to Henderson and Pingry, when these conditions were met, a problem existed for the particular individual. As Cronbach (1948, p. 34) pointed out, ". . .it is not posing the question that makes the problem, but the person's accepting it as something he must try to solve."

Textbook exercises were thus only problems if they met these necessary conditions: that is, if the student accepted the problem as his own and the solution of the problem became his goal. This did not

mean that the student must pose the problem for himself. The problem may have been just as meaningful if it was posed by the teacher or a textbook. The crucial factor was the extent to which the student's ego became involved in the problem. Therefore the value of problem solving activities could be measured in terms of the amount of involvement they demanded of the learner.

Most recent writers agreed with Henderson and Pingry in their definition of what constituted problem solving in elementary school arithmetic. According to Marks, Purdy and Kinney (1965, p. 393), a student is involved in mathematical problem solving whenever he attacks a quantitative situation calling for abilities he has not made automatic. Verbal problem solving deals with the solution of problems arising out of the application of mathematics to various aspects of life.

Fehr and Phillips (1967, p. 375) defined a verbal problem as a situation for which no ready-made procedure toward a solution is available to the person facing such a situation. Their definition was in close agreement to that of Marks, Purdy and Kinney.

Thorpe (1961) was also in close agreement with Henderson and Pingry. Her article was a good summary of the modern understanding of what constitutes a true arithmetic problem. She explained that a true problem is a situation facing individuals or a group who want to find a solution and have no ready-made method for arriving at one. A true problem is an unstructured situation. The first part of the solution of a problem is the structuring of the problem. She believed that this structuring process should begin at the physical level. Therefore, the

teaching of problem solving requires the assisting of pupils to make a transition from the physically-structured to the abstractly-structured concept. Thus, she suggested the use of concrete materials in developing problem solving ability.

II. HOW CHILDREN THINK WHEN THEY SOLVE VERBAL ARITHMETIC PROBLEMS

The answer to the question of how children think when they solve verbal arithmetic problems requires a consideration of the development of the thought processes of children. For this consideration the studies of Piaget are discussed first. Then the implications of Piaget's theory for learning in verbal arithmetic problem solving are considered.

Piaget (1951, p. 27) believed that to understand the development of knowledge, or thinking, one must first start with the idea of an operation. An operation is an internalized action which can modify an object of knowledge. For example, an operation could consist of constructing a classification, of putting things in a series, or of counting or measuring. In a speech given at Cornell University, Piaget expressed this point when he said:

To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed (Piaget, 1964, pp. 8-9).

An operation is reversible in the sense that it can take place in both directions (e.g., joining and separating). The attainment of reversibility, however, requires more than an ability to undo an observed transformation. The individual must also anticipate in thought a return to the original state. This anticipation, with its implied

annihilation of the two inverse relationships, characterizes the attainment of reversibility. An operation is never in isolation. It is always part of a total structure and is linked to other operations.

In the child's development of operational structures, which are the basis of knowledge, Piaget has distinguished four main stages. The first stage is a sensory-motor, pre-verbal stage which extends approximately through the first eighteen months of life. The second stage is a pre-operational stage which extends from about eighteen months to about seven years. The third stage is a concrete operations stage which extends from about seven years to about eleven or twelve years. Finally, the fourth stage is a formal operations stage which begins at about eleven or twelve years of age. Although the chronological ages corresponding to the stages vary a great deal from culture to culture and from individual to individual, the order of succession of the stages is constant (Flavell, 1963, p. 86).

During the sensory-motor stage, as Piaget saw it, the child is acquiring skills and adaptations of a behavioral kind. The schemas, or mental operations, of this period are sensorimotor operations. That is, they organize sensory information and result in adaptive behavior, but are not accompanied by any cognitive or conceptual representation of the behavior or of the external environment. One of the major understandings developed during this stage is the concept of object permanence (the understanding by the child that an object remains in existence even though it is not in his sensory reach (Baldwin, 1967, pp. 190-220).

With the appearance of the preoperational stage, the child begins

to experience mental pictures of the external world and of his own actions. These mental pictures are the first signs of conceptual thinking or mental operations that will eventually result in the development of logical thinking. According to Piaget these conceptual mental operations are basically different from sensori-motor operations. He explained this when he said:

First of all, there is what I call physical experience and what I call logical-mathematical experience. Physical experience consists of acting upon objects and drawing some knowledge about the objects by abstraction from the objects. . . . There is a second type of experience which I shall call logical-mathematical experience where the knowledge is not drawn from the objects but is drawn from the actions effected on the objects (Piaget, 1964, p. 27).

Like all mental activity in Piaget's system, the mental image is an internal act. It is, in fact, an internalized imitation of a specific action or event. During the pre-operational stage there is the first appearance of symbolic internalized actions that permit symbolic behavior. The child's concepts, however, are not yet coherently organized. Thus, he may arrive at contradictory conclusions when he must integrate temporally separate events.

During the stage of concrete operations the child's thinking becomes integrated into a system of operational thoughts. The child's thinking is still oriented toward the actual observation of concrete events in his environment. Now, though, his mental operations become organized into group-like structures rather than being isolated and unrelated as they were formerly.

Finally, at the age of about eleven or twelve years, the child passes into the stage of formal operations. This stage is characterized

by the development of formal, abstract thought operations. With these the child can now reason in terms of hypotheses and not only in terms of objects.

It was Piaget's opinion then, that 'thinking', or the process of mental operations, consists of internalized actions. He emphasized that these internalized actions depend on physical experience for their formation in children. This was made clear when he stated that childish thought is nearer to action than that of adults and consists simply of mentally pictured manual operations which follow each other without any necessary connection (Piaget, 1928, p. 145).

Van Engen (1961, pp. 87-92) supported the point of view that actions and manipulations are dominant in the formation of the child's concepts. He stated that thinking is merely the mental manipulation of symbols which represent concepts. He emphasized the close connection between manual operations, or actions, and the thought processes of the child. His claim was supported by demonstrating that Piaget holds that the child thinks by picturing, mentally, the manual operations that take place in a given situation. In his summary Van Engen stated that one can conclude that the perceptual, manipulatory activities are of utmost importance in the development of number concepts as well as concepts in general.

Van Engen's interpretation of Piaget's theory was supported by Almy when she wrote:

Piaget's theory leaves no question as to the importance of learning through activity. Demonstrations, pictured illustrations, particularly for the youngest children, clearly involve the child

less meaningfully than do his own manipulation and his own experimentation. While the vicarious is certainly not to be ruled out, it is direct experience that is the avenue to knowledge and logical ability (Almy, 1966, p. 137).

Churchill (1961, pp. 103-104), too, emphasized the need for concrete materials in developing childish thought. In discussing various methods of instruction in arithmetic, she stated that the child who is deprived of experience at the concrete level is being deprived of the sources of the images of which his capacity to learn depends. Thus, it is not enough for the child to see materials and watch other people operate with them. He must operate on them himself if he is to form the necessary mental operations.

Lindstedt, in contemplating the theory of Piaget, wrote:

His theories have some definite implications for the methodology of developing problem solving ability, such as: (1) Problems should be genuine and meaningful to the child. (2) "Discovery" methods should be used, provoked by relevant physical manipulation. (3) There should be a gradual and definite transition from the use of physical manipulations to the manipulation of symbols with abstract referents (Lindstedt, 1962, pp. 23-24).

Van Engen's belief that Piaget's theory has direct applications in the teaching of problem solving was demonstrated when he was asked to suggest a solution to the problem solving dilemma. Van Engen said:

It would seem that Piaget has indicated a hopeful approach to this problem. In the early stages of thought a child thinks by visualizing the manual operation which have become associated with a given group of words. Problem solving, for the child, then becomes the visualization of the actions (manual operations) suggested by the words used to convey the original quantitative situation together with the ensuing events (Van Engen, 1955, p. 362).

In another article, Van Engen (1959, p. 75) explained that the basic difference between good problem solvers and poor problem solvers

resides in differences in ability to recognize the structure of the problem. The good problem solvers know the action suggested in the problem even though they may not have been taught appropriate means for expressing this action. The ability to mentally perform the manual operations suggested by the problems leads the good problem solvers to the right answer.

In summary, as Lovell (1961, p. 44) explained, Piaget believes that mathematical concepts are developmental in nature for the child. These concepts are not derived from concrete materials themselves. Rather, they are derived from the child's appreciation of the significance of the operations performed with the concrete materials. The concepts formed by the child and his ability to manoeuvre the concepts in his mind are built up from using concrete materials.

It has been suggested that Piaget's theory of the development of childish thought has direct implications for the teaching of mathematics in the school. Van Engen, in particular, has placed much emphasis on applying Piaget's theory to the teaching of arithmetic problem solving.

III. FACTORS OF PROBLEM SOLVING ABILITY AND METHODS OF INSTRUCTION TO IMPROVE THIS ABILITY

Numerous studies have been done during the last century in an effort to determine what skills are necessary for effective arithmetic problem solving. Other studies have been designed to test what methods are most effective in developing the ability to solve verbal arithmetic

problems. In fact, Suydam (1967) stated that there were as many as eighty-four studies of arithmetic problem solving reported in various educational journals between 1900 and 1965. Only a sampling of these studies is reviewed here.

In an effort to determine what factors were most closely related to ability to solve verbal problems, Emm (1959) studied the skills of grade five boys. Through a factorial analysis she concluded that the three basic factors were: (1) a verbal factor including reading comprehension and vocabulary, (2) an arithmetic factor including computational skills and understanding of the arithmetic processes, and (3) a spacial factor including the ability to visualize and think about objects in more than one dimension. In a twin study McTaggart (1959) found these factors to be different only in emphasis for grade five girls. Girls were found to be more verbal than boys.

Corle (1958), in describing his study, concluded that there was a significant relationship between problem solving accuracy and each of the following: concept formation, reasoning ability, confidence in computation ability, and understanding of arithmetic vocabulary. Although he found the type of reasoning used by children to be highly computational in nature, he did not consider the ability to compute a major factor in problem solving ability.

Chase (1960) disagreed with Corle on this point. He concluded that the three factors of primary concern in predicting problem solving ability were: the ability to compute, the ability to read, to note details, and the possession of a fundamental knowledge of the generalizations which underlie the number system.

Blecha (1959), on the other hand, suggested that there are only two factors which determine a child's ability to cope with word problems. These are the student's mental abilities and the richness and thoroughness of his background in meanings, understandings, concepts and skills. He felt the teacher could influence ability in problem solving by developing an understanding of the technical vocabulary of mathematics. He also thought instruction in the meaning of the operations and an introduction of the problems in light of these operations would be helpful.

In summarizing the various studies reported before 1944, Johnson (1944) listed such causes for difficulty in problem solving as the following: (1) inability to read, (2) lack of method for attacking problems, (3) lack of ability in fundamentals, (4) inadequate understandings of vocabulary used in problems, and (5) carelessness in arranging the written work and lack of neatness.

It is readily apparent from the findings of the studies cited above that problem solving is a complex ability dependent on many interrelated factors.

A wide variety of instructional methods designed to improve problem solving ability has also been suggested. In discussing the studies on the improvement of problem solving done before 1944, Johnson (1944, p. 403) concluded that there was doubt concerning the relative superiority of any one method over any other well structured method of teaching problem solving. He suggested systematic and persistent training, in any reasonable procedure for attacking problems, was bound

to result in improvement. He recommended experience in solving many problems as the best way to develop problem solving ability.

Johnson (1944), himself, did a rather carefully designed study of the effect of instruction in mathematical vocabulary upon problem solving ability. He trained his experimental group in vocabulary for a fourteen-week period. This training was conducted during the regular arithmetic period while his control group used the textbook in the regular manner. He concluded that the use of instructional materials in vocabulary led to significant growth in both problem solving and understanding of specific mathematical terms.

Faulk and Landry (1961) did a similar study with sixth grade children. In their treatment the students in the experimental group had vocabulary study at the beginning of each class period. They discussed the situation presented by the problem and drew simple diagrams to demonstrate that they understood what the problem suggested. They then estimated the answer and solved the problem. When compared with the control group who had followed the Making Sure of Arithmetic Series (1952 edition), there was a significant difference at the .10 level of significance in favor of the experimental group. Faulk and Landry concluded, however, that both methods seemed to be effective.

Vanderlinde (1962) agreed with Johnson and Faulk. He found that when grade five pupils were taught eight quantitative terms per week for a period of twenty to twenty-four weeks there was significant improvement in problem solving ability when the effects of sex and intelligence were accounted for.

In an effort to test the effectiveness of writing and solving original problems as a means of improving verbal problem solving ability, Keil (1964) prepared a treatment for four grade six classes. In each lesson, during the sixteen-week treatment period, the pupils were given a verbally described situation and were asked to write and solve their own problems using the details of the situation. The Metropolitan Achievement Test was given to compare the gain in problem solving ability of the experimental group with the gain made by the control group. The pupils of the control group had spent the same amount of time doing regular textbook type problems. The results demonstrated significantly greater improvement by the pupils who had written and solved their own problems.

Pace (1961), in a similar study, used two groups of grade four pupils. Each class was given a set of twenty-four problem exercises to do. One group was continuously asked: "What do you need to solve the problem?"; and "Why would you do that?", as they solved each set of problems. The other group simply solved the twenty-four sets of problems. When the amount of improvement by the two groups was compared, the group that had been asked, "Why?", showed significantly greater improvement. Pace agreed, however, that children improve greatly in problem solving ability if they are merely given many problems to solve and then left to their own devices in solving them.

Irish (1964) attempted to determine with fourth grade children whether problem solving ability could be improved by: (1) developing the ability to generalize the meanings of the number operation and the

relations among these operations, and (2) by developing an ability to formulate original statements to express these generalizations. Her study was somewhat poorly designed, however, since only the experimental group was actively involved in the study.

Sinner (1959) also agreed with Pace in explaining that pupils need a great deal of guidance in problem solving. She suggested that problem solving should be made a part of the whole school context. She thought this would supply practice that is well-motivated and meaningful to the pupils.

The effect of group experience on improving individual problem solving ability was studied by Hudgins (1960). He hypothesized that group experiences would improve individual problem solving ability more than would individual experiences. After doing his study with fifth grade students, he concluded that although groups of students working cooperatively could solve more problems than students working alone, there was no significant improvement in verbal problem solving ability due to group experience. Hudgins also concluded that practice in specifying the steps in solving a problem did not improve performance any more than when such specification was not made.

Riedesel (1964) claimed to have demonstrated that practice in solving problems that are appropriate in level of difficulty for each pupil will result in greater improvement in problem solving ability. He provided verbal problems of two levels of difficulty for use in the same classroom. Each class was then taught using the following highly recommended procedures: (1) writing the number question or mathematical

sentence, (2) using drawings and diagrams, (3) having pupils formulate original problems, (4) presenting problems orally, and (5) using problems that do not contain numerals. After thirty problem solving lessons the gain in problem solving ability of the students using the two levels of problems was compared with the gain of students following typical textbook instruction. There was a significant difference in mean gains in favor of the students using the two level problems. It should be noted, however, that it was difficult to determine whether the gain in problem solving ability was a result of the use of two levels of problems or a result of the procedures used in the instruction of the experimental group.

In general, the research work on techniques for teaching problem solving has had differing results. No optimum technique has been evolved and the variety of possible techniques has been enlarging rather than narrowing. Koenker (1958) listed twenty successful methods of improving problem solving ability that had been developed by the year 1956.

While the search for more effective methods of teaching problem solving ability continues, there has been a change in the proposed purpose for developing problem solving ability. Lindstedt made this point clear when he wrote:

We note a recent change in the asserted objectives of arithmetic instruction. During the decades of the 1930's and 1940's many arithmetic educators stressed the acquisition of computational skill and its application to the solution of problems, as primary objectives. . . .

During the last decade, however, there has been a realization

that there is a more basic objective for arithmetic instruction. . . .This is the understanding of mathematical concepts "per se," as a discipline of related, organized ideas (Lindstedt, 1962, p. 2).

Thus, there has been a recent change in research to the study of the effectiveness of stressing the action-sequence structures of arithmetic problems as a technique for developing problem solving ability.

Lindstedt (1962), himself, designed a study to test the strengths of a program of instruction that stressed the action-sequence in problems. He hoped to determine the effectiveness of this program in developing patterns of thought and in improving problem solving ability. He concluded that the technique which stressed action-sequence was more effective in developing problem solving ability than was the traditional method.

Lerch and Hamilton (1966) tended to support Lindstedt in his claims. They attempted to compare the growth in problem solving of pupils who studied a structural-equation approach and those who studied the traditional: "What is given?", "What is asked?", "What process should be used?", approach. They came to the conclusion that those students who were in the structure-equation group were significantly better at structuring the problem but there was no difference in ability to perform the necessary computations.

Wilson's (1967) results strongly disagreed with those of Lerch and Hamilton. His study suggested that children in the wanted-given program are significantly more successful in choosing the correct operation than those who are instructed in recognizing action-sequence

structures. His conclusions, however, were severely criticized by Zweng (1968) as being misleading on the basis of the basic objectives of arithmetic instruction. She suggested that his study may have given a distinct advantage to the students studying the traditional approach since those in the action-sequence instruction may not have had sufficient time to adjust to the new approach.

IV. THE USE OF MANIPULATIVE MATERIALS IN TEACHING PROBLEM SOLVING

There has been little significant research done on the use of manipulative materials in the teaching of verbal arithmetic problem solving. Much has been written, however, to encourage the use of manipulative materials in the teaching of arithmetic.

As early as 1935, in the tenth National Council of Teachers of Mathematics Yearbook, Brownell called for a change in the teaching of arithmetic. He emphasized the need for manipulative materials to give the pupil meaningful experiences with "concrete number." He wrote:

The child begins with concrete number--with objects which he can see and handle. He makes groups of objects, compares groups of objects, estimates the total in given groups of objects, learns to recognize at a glance the number of objects in small groups and in larger groups when the latter are in regular patterns. Eventually he comes to think of concrete numbers in terms which are essentially abstract (Brownell, 1935, p. 22).

In the sixteenth yearbook, Sauble (1941, pp. 157-58) concluded that materials used in the teaching of arithmetic contributed to the mathematical meaning and to the social significance of arithmetic. She urged teachers to use their originality and initiative in devising materials that would enrich the teaching of arithmetic.

By 1945 the emphasis on the use of manipulative and other instructional materials was clearly shown by the dedication of the eighteenth yearbook of the National Council of Teachers of Mathematics to a discussion of multi-sensory aids in the teaching of mathematics. In this yearbook various devices and types of materials were described and their use was recommended to all teachers.

Grossnickle, Junge and Metzner (1951, pp. 155-56) pointed out that arithmetic learning consists of an orderly series of experiences which begins with concrete objects and progresses through abstractions. They stressed the importance of manipulative materials in the early stages of the development of mathematical concepts. They also emphasized the need for variety in the materials being used.

Finally, in 1963 the use of manipulative materials was emphasized by the twenty-seventh yearbook of the National Council of Teachers of Mathematics in its discussion of enrichment mathematics. The use of manipulative materials during the introductory stages of instruction was particularly emphasized.

Most of the research which investigated the use of manipulative materials in teaching problem solving has been concerned with commercial materials such as Cuisenaire rods and the multi-basic blocks designed by Dienes. A brief discussion of these materials will be presented here in preparation for this study.

The Cuisenaire rods were designed in an attempt to supply concrete manipulative materials which would provide a basis for the pupil's learning of mathematical concepts and to assist him in discovering

important mathematical principles and relationships. Since the solving of verbal arithmetic problems requires an understanding of various mathematical relationships, improvement in problem solving ability has been suggested as one of the results of using Cuisenaire materials. The promoters of these materials suggest that provision for the teaching of problem solving can best be made if the skills of computation are developed before these skills are applied to the solution of verbal arithmetic problems (Gattegno, 1962, pp. 63-65).

Since their introduction into Canadian classrooms, much research has been done to determine the effectiveness of Cuisenaire materials in developing various basic mathematical concepts. In an article prepared for the Canadian Council of Research in Education, Nelson (1964) conceded that the Cuisenaire Method was effective in developing computational skill on the part of elementary school children. However, on the topic of problem solving he pointed out that, in some of the studies made, pupils in the Cuisenaire program did not perform as well as did pupils using other programs of instruction. He suggested that this weakness may have been due to a lack of emphasis on problem solving rather than an inherent weakness of the materials. No definitive decision was made on the effectiveness of Cuisenaire rods in teaching problem solving but further research was suggested.

Using the theories of Piaget as a basis, Dienes has designed his multi-basic blocks in an attempt to prepare manipulative materials that will effectively develop basic mathematical concepts. Although no direct reference is made to teaching verbal arithmetic problem solving,

Dienes has found that children can be taught various mathematical concepts using his materials. He claimed to have led normal ten-year old pupils to understand the solution of linear and quadratic equations, as well as factorization of quadratic functions (Dienes, 1959, p. 28).

Dienes warned, however, that if pupils learn to abstract a concept from just one type of manipulative material there is danger of producing a perceptual block. He explained that confusion may result because pupils may not be able to understand that principles or properties discovered from one type of material are applicable to other situations (Dienes, 1966, p. 53). Thus, Dienes recommended the use of a variety of situations and manipulative materials to teach mathematical concepts.

V. SUMMARY

Any statement about how children think and how ability in problem solving can be improved must be made with caution since the results of research are often inconclusive and sometimes conflicting. However, some trends and generalizations seem to appear that may be summarized with a reasonable degree of confidence.

Modern writers have suggested that the structuring of the verbal arithmetic problem is the main aspect of its solution. Thus, the modern emphasis is on aiding the pupil to develop symbolic structures for the physically structured situations described by the written problems.

That the structuring of the verbal arithmetic problem is basic was suggested by a brief study of the research and theory of Jean Piaget.

The interpretations of his theory, as made by Churchill and Van Engen, demonstrated the need for active participation by the child. Churchill, in particular, has emphasized the need for concrete materials in developing the thought processes required for effective mathematical problem solving.

From the discussion of methods for improving problem solving it seems evident that it is possible to bring about substantial improvement in the ability of pupils to solve verbal arithmetic problems. Also, since most investigations reported gains by most groups, it would appear that supplying pupils with many opportunities to solve problems is a good method of improving problem solving.

There is some indication that instruction which stresses the action-sequence structuring of problems is a satisfactory method of improving problem solving ability. This is particularly true if we accept one of the most recently stated objectives of the mathematics program, that is, to develop ability on the part of the child to represent the physical world by mathematical models.

As a result of this renewed emphasis on the representation of the physical world, there has been an increased emphasis on the use of concrete materials to aid the pupil's learning. This was clearly shown in the statement made by the Cambridge Conference on School Mathematics:

Whether one thinks in terms of the pre-mathematical experiences that are embodied in the manipulation of physical materials, whether one regards these physical objects as aids to effective communication between teacher and child, or whether one regards them as attractive objects that increase motivation, the conclusion is inescapable that children can study mathematics more satisfactorily when each child has abundant opportunity to manipulate suitable physical objects. . . (Cambridge Conference, 1963, p. 33).

CHAPTER III

THE EXPERIMENTAL DESIGN AND STATISTICAL ANALYSIS

I. A DESCRIPTION OF THE PROCEDURES

The purpose of this study was to determine the effectiveness of the use of manipulative materials in developing ability to solve additive and subtractive verbal arithmetic problems.

Eight classes of grade three pupils were selected for this study. Four of the classes were assigned to the control group and the other four classes formed the experimental group. The classes forming the experimental group and the control group were compared on the basis of age, socio-economic status, intelligence, reading comprehension ability, ability to compute, and ability to solve verbal arithmetic problems. These comparisons were made to determine if the experimental and control groups could be considered representative of the same population.

At the beginning of the study, the subjects in the control group and in the experimental group were tested to determine their ability to solve additive and subtractive verbal arithmetic problems. Both groups were then given a treatment consisting of eight lessons of instruction designed to develop ability to solve verbal arithmetic problems. An attempt was made to keep the treatments the same for both groups in every detail except that the experimental group used a variety of manipulative materials in their discussion of the verbal problems. The

control group used no manipulative materials of any type during the treatments.

Following the treatments the ability of the subjects to solve verbal arithmetic problems was tested again using the same test as before the treatments. The amount of gain in ability to solve verbal arithmetic problems that could be attributed to the treatments was computed. The results for the experimental and the control groups were compared to determine if the use of manipulative materials resulted in significantly more gain in ability to solve verbal arithmetic problems.

To investigate further the effectiveness of the use of manipulative materials in teaching problem solving, the results from various subgroups of the total group of subjects were considered separately. Comparisons were made between the results of the experimental group and the control group for the following subgroups: all boys, all girls, pupils of high intelligence, pupils of average intelligence and pupils of low intelligence. The same comparisons were made for pupils of high socio-economic status and low socio-economic status.

After a one-month interval the test was administered a third time to determine if the use of manipulative materials had any effect on the retention of the ability to solve verbal arithmetic problems. The data from this post test were analyzed for each of the subgroups.

II. SELECTION OF THE CLASSES

Ten classes of grade three pupils from six different schools in the Edmonton Separate School System were recommended for participation

in this study. From these ten classes, eight classes, containing 248 pupils, were selected. Of the eight classes selected, four were chosen to form the experimental group which used manipulative materials during the treatment. The other four classes were used as the control group.

The eight classes that were selected came from four different schools situated in south and west Edmonton. In two of these schools courses were offered for grades 1 to 6. In the other two schools courses were offered for grades 1 to 9.

The four schools were selected for participation in the study, and the eight classes were assigned to either the experimental or control group on the following basis:

1. Each school contained only two classes of grade three students. In two of the schools the students had been assigned randomly to their classes. In the other two schools the students were grouped according to ability. In each of those schools where ability grouping was used, one class included pupils of average and below average ability. The other class included pupils of average and above average ability as determined by the pupils' school achievement in the previous year.

One class from each school was assigned to the experimental group and the other class was assigned to the control group. Care was taken to insure that both the experimental and control groups contained classes of similar ability as described by the class teachers.

2. Each class contained approximately the same number of pupils. Although Spitzer's (1954) study of 148 third and sixth grade classrooms showed class size was not a significant factor in achievement, an attempt

was made to keep class size as equal as possible. Table I shows a summary of the number of students enrolled in each classroom during the treatments.

TABLE I
SIZE OF THE PARTICIPATING CLASSES

Experimental Group		Control Group	
	31		27
	32		31
	33		32
	31		31
Total	127		121

The average class size for the experimental group was 31.8 and the average class size for the control group was 30.2.

3. Each class used the grade three textbook of Seeing Through Arithmetic (1959 edition) series. The series they were studying had been in use in each of the participating schools for at least three years.

The participating teachers were interviewed to determine if the arithmetic experiences of any class was greatly different from that of the other classes during the time previous to the study. Each teacher claimed to follow the guidebook designed for use with the Seeing Through Arithmetic series in her arithmetic instruction. No serious differences

in arithmetic experiences between the classes were apparent.

4. The amount of class time spent in the formal instruction of arithmetic was approximately the same in each classroom. Each teacher taught arithmetic for thirty-five to forty minutes per day.

5. During the interview with each participating teacher an effort was made to determine if any class had an appreciably greater amount of experience with concrete materials in their arithmetic lessons previous to the beginning of the experiment than did the other classes. Each teacher claimed to use concrete materials only occasionally for demonstration purposes. None of the teachers had used concrete materials of the type, or in the manner, used in this study.

III. TREATMENTS

Since this study was designed to investigate the effectiveness of using manipulative materials in the instruction of arithmetic problem solving, the use of these materials was the treatment variable.

Preparation of the Lessons

Eight problem solving lessons (see Appendix A), were prepared for use in the treatments. These lessons were designed for use by both the experimental group and the control group.

Each lesson consisted of ten verbal arithmetic problems. The problems were carefully written to avoid any ambiguity and to insure that the question being asked was clearly understood. The format of each problem was based on that used by the revised Seeing Through

Arithmetic series (1967 edition). This format was used because it was most convenient and clearly outlined the steps necessary for the completion of each problem. Using this format each problem was designed so the pupil would: (1) read the problem, (2) select or write the equation that best illustrated the action being suggested by the problem, (3) compute to find the unknown term in the equation, and (4) write the numeral in the sentence that answers the question being asked by the problem. The lessons were duplicated so that each pupil had his own copy.

Since the experimental group was to use manipulative materials in demonstrating the structure or 'action' of each problem, the numbers used in the problems were kept small. No problem contained a number larger than forty-eight. This made the computations relatively easy but should not have altered the effectiveness of the lessons.

In accordance with the classification of Hartung, et al. (1960) only six types of problems were included in the lessons. According to Hartung, problems can be classified in two different ways. Problems can be classified according to the situation suggested by the problem and according to the computational process which is required for its solution. Table II contains an illustration of the six types of problems included in the lessons. The table also shows the grade when each type is first introduced by the Seeing Through Arithmetic series (1959 edition) and the lessons in which each type of problem was represented.

It should be noted that four of the types of problems included in the lessons normally have been introduced to the students by the end

TABLE II
THE PROBLEM TYPES INCLUDED IN THE LESSONS

Situation-Process Description	Equation showing structure	Computa- tional process	Grade intro- duced	Lessons in which the problem type occurred
Additive-addition	$8+5 = n$	$8+5$	3	1, 3, 7, 8
Additive-subtrac- tion	$8+n = 13$	$13-8$	3	2, 3, 5, 6, 7, 8
Additive-subtrac- tion	$n+5 = 13$	$13-5$	3	2, 3, 5, 6, 7, 8
Subtractive- subtraction	$13-5 = n$	$13-5$	3	1, 3, 7, 8
Subtractive- subtraction	$13-n = 8$	$13-8$	4	4, 5, 6, 7, 8
Subtractive- addition	$n-5 = 8$	$8+5$	4	4, 5, 6, 7, 8

of grade three. These types of problems had been studied by the pupils before the treatments began. The other two types of problems are not normally introduced by the end of grade three. These types were included in the treatments to allow for a greater margin of growth.

Lessons 1, 2, and 4 were designed to introduce each of the six types of problems being studied. Lessons 3, 5, 6, 7, and 8 were designed to give the pupils more practice in solving problems of the various types.

The Concrete Materials

Each pupil in the experimental group was supplied with a set of

concrete materials. Each set contained three types of materials. These materials included fifty colored counting sticks two and one-half inches in length; fifty tile squares with dimensions of one-half inch; and fifty rectangles, cut from heavy ticketboard, with dimensions two inches by one-half inch. For convenience in storing all the materials were supplied to the pupil in a small container. Figure 1 shows a set of concrete materials.



FIGURE 1

THE CONCRETE MATERIALS

These concrete materials were selected for the study because of their convenience in handling and variety in shape and color. Syer (1961, pp. 104-08) has indicated that the main function of the use of concrete materials is to motivate the learner to structure the problem so that the problem is more clearly defined. Thus, no attempt was made to supply

the exact materials described in each of the problems. To avoid the possibility of the pupils attaching any numerical significance to one type of material, objects representing a variety of colors, shapes, and sizes were supplied.

Procedure of the Treatments

The procedure for each lesson was carefully outlined, in written form, for each teacher (see Appendix B). An effort was made to control as many variables as possible. The only difference in the procedure for the two groups was that the experimental group used concrete materials.

During the first four lessons the pupils in the control group read each problem silently. Then they listened as one member of the class read the problem aloud. The pupils were then asked to explain what action was suggested by the problem. After the pupils had discussed the action they were told to select or write the arithmetic sentence that would best represent the action suggested by the problem. Next the pupils were directed to compute to find the unknown term in the arithmetic sentence. The correct process of computation was then discussed to insure that all the pupils understood whether they should add or subtract. When the pupils had completed the computation, one member of the class was asked to write both the correct arithmetic sentence and the computation on the blackboard. The pupils were asked to verify their own answer by the example on the blackboard and place the correct numeral in the sentence which answered the question asked by the problem. The same procedure was used for each problem in the first four lessons.

The procedure for lessons 5, 6, 7, and 8 varied only slightly from

that used in the first four lessons. In lessons 5 and 6 the pupils were asked to complete the first five items by themselves before they began to discuss the action suggested by the problem and to verify their answers. This procedure gave the pupils more independence in solving the problems and allowed the pupils to work at their own rate. In lessons 7 and 8 the pupils were directed to complete all ten problems before the discussion and verification began.

The pupils in the experimental group had the same series of verbal problems but they studied them in a different manner. During the first four lessons each problem was read silently by the pupils to insure that it was clearly understood. Instead of merely discussing the action suggested by the problem, the pupils in the experimental group were asked to manipulate objects from their own set of concrete materials to demonstrate the action. One member of the class was then asked to demonstrate to the class the action described by the problem. He did this using concrete materials. Once the pupils clearly understood the structure of the problem, they were asked to select or write the correct arithmetic sentence. They then computed to find the unknown term in the arithmetic sentence. If the pupils encountered any difficulty in deciding what computational process they should use, they were encouraged to use their concrete materials to discover the correct process. One pupil was then asked to show his work on the blackboard. The pupils then verified their own answers, completed the statement and prepared for the next item.

In lessons 5, 6, 7, and 8, the pupils in the experimental group

followed a similar procedure but an attempt was made to allow them to work more independently than in the first four lessons. In lessons 5 and 6 the pupils were directed to do the first five problems by themselves before they began to demonstrate and discuss the action suggested by each problem. In lessons 7 and 8 the pupils did all the problems independently then they discussed and demonstrated the action suggested by each problem. In this way an attempt was made to help the pupils become less dependent on the concrete materials during the later lessons.

Control of the Treatments

In an effort to keep the procedure used by each teacher as similar as possible, the investigator met with each teacher at regular intervals during the treatments. During these meetings the procedure of the lessons was discussed and the teachers were encouraged to follow the directions for each lesson as carefully as possible. Any misunderstandings concerning the interpretation of the directions were discussed with the teacher.

Special attention was given to the length of the lessons since no valid conclusions could be drawn if one group of students spent an appreciably greater length of time on the lessons than did the other group. To correct for this possibility the teachers were asked to maintain the length of each lesson between the thirty-five to forty minute time margin. In some of the lessons the teachers in the experimental group were forced to omit as many as three of the problems because of lack of time.

During the study each teacher was asked to record the time when her arithmetic instruction was started and completed. These records were checked to determine if any of the lessons were given at widely differing times of the school day. This was done since there is a possibility that physical fatigue could have an effect on the effectiveness of the treatments. All of the lessons were reported to have been taught during the first two hours of the school day so the possibility of fatigue should have been eliminated.

Each teacher was asked to refrain from any instruction in problem solving, other than that supplied by the prepared lessons, during the interval from the pretest to the post test of the study. This was done because additional instruction in problem solving could cause an imbalance in the amount of instructional time the control and experimental groups were given. To aid the teachers to comply with this request, sections of the textbook dealing with topics that included no problem solving were suggested to them.

The teacher variable may be considered a major determinant of the effectiveness of any type of instruction. For this reason the teachers participating in this study were compared with regard to amount of training and amount of experience. While these are not the only criteria that determine the effectiveness of a teacher, these aspects may be expected to be basic to teaching skill. Table III contains a summary of the amount of training and years of experience possessed by each of the participating teachers.

On the basis of evidence reported by Ellena, Stevenson, and Webb

TABLE III
AMOUNT OF TRAINING AND EXPERIENCE OF TEACHERS IN THE STUDY

Experimental Teachers		Control Teachers	
Years of Training	Years of Experience	Years of Training	Years of Experience
4	3	3	3
1	7	1.5	9
2	2	2	1
3.8	6	4	4
Average 2.70	4.50	2.62	4.25

(1961, pp. 22-25), the teachers of the control group and the experimental group may be considered equally competent for this study. These authors report that teachers make their greatest professional growth during the first four years of their experience, reaching a plateau about the fifth year. They also report significant differences in teacher competence due to type and amount of teacher training in favor of those teachers with the greater amount of professional training.

Schedule of the Study

In order to avoid differences between the groups in the duration of the treatments and to maintain a uniform sequence for the administration of the various tests, all the participating classes followed the same schedule. The schedule for the experiment was as follows:

1. During the week of May 6 to 10 - Reading Test

2. May 13 - Problem Solving Test (Pretest)
3. Treatment - May 14 - Lesson 1
May 15 - Lesson 2
May 16 - Lesson 3
May 17 - Lesson 4
May 21 - Lesson 5
May 22 - Lesson 6
May 23 - Lesson 7
May 24 - Lesson 8
4. May 27 - Problem Solving Test (Post test)
5. May 28 - Test of Computational Ability
6. May 28 - Questionnaire concerning mathematical experiences
at home
7. June 24 - Problem Solving Test (Retest)

IV. THE TESTING AND CLASSIFICATION INSTRUMENTS

An effort was made to control or account for any significant differences that may have existed between the control group and the experimental group at the beginning of the study. For this reason comparisons of the classes were made on the basis of the socio-economic status of the students, rate of promotion in earlier grades, and arithmetic experiences of the pupils. Further comparisons were made of the intelligence and reading comprehension ability of the participants.

Standardized Instruments

The Blishen Occupational Class Scale (Blishen, 1961) was used as the instrument for comparing the socio-economic status of the subjects. The occupation of the main wage earner is rated on this scale according to a variety of characteristics which include income and years of training. In this study the occupation of the father was used as the main criterion. In the case where the father was deceased or unemployed, the occupation of the mother was considered.

Each pupil's intelligence was measured by the Otis Quick-Scoring Mental Ability Test: Alpha Test, Form A during the month of December, 1967. These tests were administered and scored by the classroom teachers. The resulting intelligence quotients, as recorded in the cumulative records, were used for this study. Although this test is reported by the author (Otis, 1954, p. 13) to have a reliability of .87, Yates (1958) states that the results of such a test should be interpreted cautiously if used to predict school achievement.

Reading comprehension ability repeatedly has been suggested as a factor of problem solving ability (LaZerte, 1933, p. 123). To control for this variable a score representing the reading ability of each pupil was obtained using the Reading Test from the Metropolitan Achievement Tests: Elementary Battery (Durost, et al., 1959). This test contains forty-four items and requires twenty-two minutes to administer. Robinson (1965) described this test as one of the best survey tests of reading achievement on the market today. Its reliability was reported as .90 by the test designers (Durost, et al., 1959).

Questionnaire Concerning Promotion and Arithmetic Experiences

The rate of promotion in the earlier grades and the pupils' experiences with arithmetic problems outside the classroom may be expected to have some bearing on the outcome of this investigation. In an effort to control these variables the pupils were asked to complete a questionnaire (see Appendix C) designed to give some indication of the extent of their experiences.

The first ten questions of the questionnaire concerned number experiences frequently encountered outside the classroom. The content of these questions closely agreed with those listed by Keil (1964, pp. 149-52) in her study of problem solving ability, and by Willey (1943) in his study of the use of arithmetic.

The last question on the questionnaire was designed to determine the rate of promotion of each pupil. This question asked the pupil to indicate how many years he had been at school.

In an effort to insure that the pupils understood the questions contained in the questionnaire, they were presented orally. The questions were read and explained to each class so they understood how to indicate their answers.

Development and Description of the Problem Solving Test

Since the primary purpose of this study was to investigate the effectiveness of using concrete materials in the instruction of problem solving, and since there was no published test specially designed to test ability to solve additive and subtractive problems at the grade three level, a test (see Appendix D) was designed for use in this study.

It was considered important that the test measure both the pupil's ability to express the structure of the problem in the form of an equation and the pupil's ability to arrive at a correct numerical answer for the problem. It was also considered important that the problems be of the same types and use the same terminology as the problems contained in the Seeing Through Arithmetic series of textbooks since this was the series used by the pupils. For these reasons the test format was planned so the pupils would: (1) read the problem, (2) write an arithmetic sentence, or equation, to represent the structure suggested by the problem, (3) compute to determine the unknown term in the equation, and (4) write the correct numeral in the sentence which answered the question asked by the problem. The format of this test was the same as that of the lessons used during the treatment in the study.

Following the classification of Hartung, et al. (1960) the test consisted of twenty-four items designed to estimate the pupil's ability to solve six types of additive and subtractive problems. Each type of problem was represented by four different items in the test. Table IV lists the type of problem represented by each item on the test.

TABLE IV

THE TYPES OF PROBLEMS INCLUDED IN THE PROBLEM SOLVING TEST

Situation - Process Description	Equation Showing Structure	Test Items Represent- ing Each Problem Type
Additive - Addition	$8 + 4 = n$	4,6,19,22
Additive - Subtraction	$8 + n = 12$	7,8,10,15
Additive - Subtraction	$n + 4 = 12$	2,13,20,11
Subtractive - Subtraction	$12 - 4 = n$	3,5,14,16
Subtractive - Subtraction	$12 - n = 8$	1,17,18,23
Subtractive - Addition	$n - 4 = 8$	9,12,21,24

Scoring the Problem Solving Test

A study by Lerch and Hamilton (1966) suggested that the ability to structure the problem and the ability to arrive at a correct numerical answer are both related to success in problem solving. They found, however, that competence in structuring the problems did not necessarily mean that the pupils were competent in arriving at correct numerical answers. On the basis of their evidence, and for the purposes of this study, the Problem Solving Test was scored in two separate ways. On the first scoring only the pupil's ability to write equations expressing the action suggested by each of the problems was considered. This score represented the pupil's ability to structure the problems. On the second scoring only the number of correct numerical answers was considered. This score represented the pupil's ability to select the

necessary computational process and to compute correctly.

The Pilot Study

A pilot study was conducted to determine the difficulty, and an appropriate length of testing time, for the Problem Solving Test. Two of the classes which had been recommended for participation in the main study, but were not selected, took part in the pilot study. These classes were in two different schools and contained sixty pupils.

After the test items were carefully written to avoid any ambiguity of problem type, they were criticized by the teachers participating in the pilot study. On the basis of these criticisms, some of the items were rewritten. The test was then duplicated and administered to the pupils in the pilot study. The teachers were asked to appraise the test again and to list any words that the pupils indicated they could not read or understand. Both teachers agreed that the items were satisfactory and none of the words were considered too difficult.

The reading level of each word in the test was also determined using the Buckingham and Dolch Combined Word List (1936). Since all the words were shown to be within the grade three reading level, no revisions were made.

The tests from the pilot study were scored to determine a satisfactory length of testing time and to determine whether the test was of sufficient difficulty to allow for a reasonable margin of improvement. On the basis of these results it was decided that the thirty minutes which had been allotted was adequate for most pupils to complete the test. The results also showed the test to be of sufficient difficulty

to allow for a reasonable margin of improvement.

The Problem Solving Test was administered three times during the study. It was used before the treatments then after the treatments, and again about one month later. Because it was administered three times it is important that this test be reasonably valid and reliable as an instrument for measuring ability to solve verbal arithmetic problems.

An argument for the validity of the Problem Solving Test is that it actually contains verbal arithmetic problems of each of the six types which were considered in the study.

The test reliability was calculated using the computer facilities at the University of Alberta. Using the TES100 computer program, the Kuder-Richardson Formula 20 reliability index was determined. The KR-20 is a measure of internal consistency or homogeneity of the test items. If the test items have high interrelations then the reliability coefficient will be high. If the interrelations are low because the test items measure different attributes then the reliability coefficient will be low (Ferguson, 1959, pp. 280-81).

Since the KR-20 method of calculating reliability is considered inappropriate for tests where speed is a major determinant of success (Thorndike and Hagen, 1962, pp. 181-82), the post test results were used to determine the reliability coefficient. In this round of testing eighty-six per cent of the pupils completed the test in the allotted time. Thus, the reliability coefficients given here are higher than if all the pupils had completed the test. Using the post test data, the

calculated KR-20 reliability index was .83 when the scores representing ability to structure the problem were considered. When the scores representing ability to arrive at correct numerical answers were considered, the reliability index was calculated as .83 also.

Preparation of the Computation Test

Ability to compute accurately has been shown to be correlated to ability to solve verbal arithmetic problems (Chase, 1960). No suitable tests were available that measured only the pupil's ability to add and subtract, so an Arithmetic Computation Test (see Appendix E) was developed for this study.

The Arithmetic Computation Test was designed to contain twelve addition and twelve subtraction items. Care was taken to insure that the skills of regrouping which are required to add and subtract accurately were thoroughly tested. The test was designed to be a power test rather than a test of speed since computational speed was not being investigated in this study.

The test was administered to the group of sixty students in the pilot study to determine if it was too difficult for grade three pupils. From an analysis of the results the test was found to have a mean of 19.7 and a standard deviation of 3.17 for the pilot study. The reliability of the test was calculated using the Kuder-Richardson Formula 21. This formula was used because it is simply calculated and yields a reasonably close approximation to the Kuder-Richardson Formula 20 (Thorndike and Hagen, 1962, p. 181). The calculated reliability coefficient was .68. On the basis of these data the Computation Test

was considered adequate for use in this study.

V. STATISTICAL ANALYSIS OF DATA

After attempting to demonstrate that the subjects included in the control group and the experimental group were representative of the same population, analysis of covariance by multiple linear regression was used to test each null hypothesis. This analysis makes it possible to take into account initial differences between the subjects.

All the data were processed on the I.B.M. 360/67 computer at the University of Alberta using the (REG200) Regression Analysis for Analysis of Variance program.

Analyses were performed for each set of data obtained by scoring the Problem Solving Test in the two ways described earlier. For each analysis pretreatment problem solving ability, reading comprehension ability and computational ability were used as covariates. The analyses were performed using successively all subjects, girls, boys, high intelligence groups, average intelligence groups, and low intelligence groups. The same analyses were done using the data of high socio-economic status groups and low socio-economic status groups. An F-ratio test for each of the sources of variation was then computed and analyzed at the .05 level of significance.

CHAPTER IV

ANALYSIS OF DATA AND RESULTS

This study was designed to investigate the effectiveness of using certain manipulative materials for the teaching of verbal problem solving in grade three arithmetic.

Before considering the data analysis, it should be noted that the design used in this study made it impractical to use random sampling in assigning the subjects to the control and experimental groups. Total school classes were chosen and randomly assigned to the groups. Any statistical inferences must be made on the basis of this limitation.

In the first part of this chapter several aspects of the control and experimental groups will be considered. By considering the scores from the various tests an attempt will be made to demonstrate that the control group and the experimental group are sufficiently similar to allow for statistical inference. In the second part of this chapter, each of the hypotheses will be considered. The data will be analyzed and the results of the analysis outlined. Finally the results of the tests of the hypotheses will be summarized.

I. COMPARISON OF THE EXPERIMENTAL AND CONTROL GROUPS

During the experiment some pupils were absent when the lessons were taught or when the tests were administered. Pupils who missed more than one treatment lesson or any of the tests were not included in

the study. Fifteen pupils from the original sample of 248 pupils were excluded from the study because their data were incomplete. Table V shows the number of pupils in each class involved in the statistical analysis.

TABLE V
NUMBER OF PUPILS IN EACH CLASS WHOSE DATA WERE
USED IN THE ANALYSIS

	Experimental Group	Control Group
	28	27
	31	31
	28	27
	30	31
Total	117	116

All of the following statistical analysis is based on the 117 pupils in the experimental group and the 116 pupils in the control group.

The socio-economic status of the pupils in the control and experimental groups was rated using the Blishen Occupational Class Scale. A summary of the number of pupils in each of the socio-economic classes is shown in Table VI.

In 1951, when the Blishen Occupational Class Scale was developed, the mean for the Canadian population was 50 and the standard deviation was 10 (Blishen, 1961). Thus, the pupils in both the control and the

TABLE VI
SOCIO-ECONOMIC STATUS OF PUPILS IN EACH GROUP¹

Socio-economic class	Numerical rating	Frequency	
		Experimental	Control
Class 1	73.2 - 90.0	13	9
Class 2	57.0 - 72.9	49	45
Class 3	52.0 - 56.9	8	6
Class 4	50.5 - 51.9	10	2
Class 5	45.1 - 50.4	19	40
Class 6	48.1 - 45.0	12	3
Class 7	32.0 - 41.7	6	11
Mean rating		57.05	54.49
Variance		126.56	98.80
Standard deviation		11.25	9.94

¹ As rated by the Blishen Occupational Class Scale.

experimental group had a mean socio-economic score somewhat higher than the average for the Canadian population in 1951. It should be noted also that none of the pupils in either group had a rating of less than 40.0 on the socio-economic scale.

A t-test was used to compare the difference in the means for the experimental and control groups. Although the mean for the experimental group was higher than that of the control group, the difference was found to be not significant.

The amount of pupils' experience with numbers outside the classroom was also compared. A questionnaire was used in an attempt to determine if there was an appreciable difference between the experimental and control groups' non-school number experiences. The responses to the questionnaire are summarized in Table VII. The per cent of affirmative and negative responses by each group is also shown.

TABLE VII
ARITHMETIC EXPERIENCES OUTSIDE THE CLASSROOM¹

Experience	Experimental				Control			
	Frequency		Per Cent		Frequency		Per Cent	
	Yes	No	Yes	No	Yes	No	Yes	No
Allowance	93	24	79	21	95	21	82	18
Bank account	40	77	34	66	38	78	33	67
Selling to earn money	100	17	85	15	107	9	92	8
Play number games	110	7	94	6	111	5	96	4
Keeping score	96	21	82	18	102	14	88	12
Use recipes	69	48	58	42	76	40	66	34
Buying things alone	112	5	96	4	104	12	89	11
Using measurements	90	27	77	23	91	25	78	22
Hobby using numbers	49	68	45	55	72	44	62	38
Total	842	328	73	27	862	298	74	26

¹As tabulated from the questionnaire.

The per cent of affirmative responses by each group was similar for each of the number experiences considered except for the use of

numbers in a hobby. There was a difference of only one per cent between the total 'yes' responses of the control and experimental groups.

The rate of promotion may have had an effect on the amount of arithmetic experience the pupils in each group had. From the questionnaire the number of pupils who had repeated a grade was determined. The findings of the questionnaire are summarized in Table VIII.

TABLE VIII
THE NUMBER OF PUPILS WHO HAD REPEATED A PREVIOUS GRADE

Grade repeated	Experimental	Control
One	1	5
Two	3	3
Three	1	0
Total	5	8

None of the pupils participating in the study had failed more than once. The pupils were included in this study regardless of whether they had been retained at any grade level.

The ages of the pupils participating in the study may have had an effect on the amount of improvement made by the members of the control and experimental groups. The pupils' ages are shown on the histogram in Figure 2.

The ages of the pupils ranged from 8 years 2 months to 10 years 1 month for the experimental group. For the control group the ages of

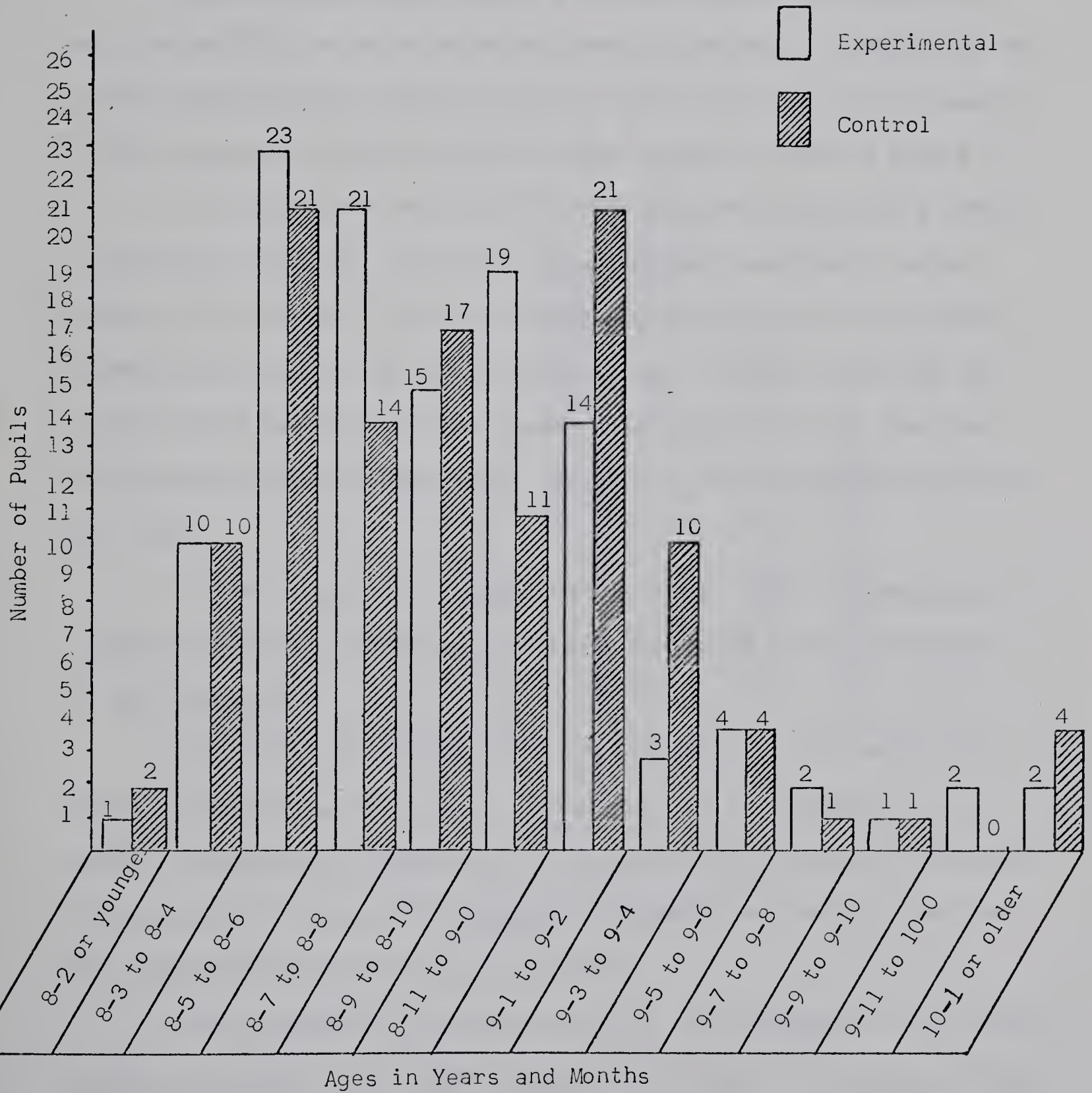


FIGURE 2

AGES OF THE PUPILS IN THE EXPERIMENTAL AND CONTROL GROUPS

the pupils ranged from 7 years 8 months to 10 years 5 months. The median age was 8 years 9 months for the experimental group and 8 years 10 months for the control group.

Reading comprehension ability has been shown to be correlated with the ability to solve verbal arithmetic problems. A summary of the reading comprehension ability as shown by the raw scores of the pupils in the experimental group and the control group is shown in Figure 3.

The histogram shows that the reading scores approached a normal distribution for both the control group and the experimental group. However, the scores of the control group are slightly more negatively skewed than those of the experimental group. The mean score for the control group was 27.38 with a standard deviation of 6.70. For the experimental group the mean score was 25.11 and the standard deviation was 7.09.

A t-test was used in comparing the means. With 232 degrees of freedom a t-ratio of 6.28 was calculated and found to be significant at the .05 level.

The ability of the pupils to add and subtract was tested. The tests were scored and the results were compiled to determine if the pupils in either the experimental or control group were more proficient in the ability to compute accurately. A summary of the raw scores on the Computation Test is shown in Figure 4.

The histograms of the scores of both the experimental and control groups are skewed to the right suggesting that most of the pupils found the test easy. Only three pupils in the experimental group and one

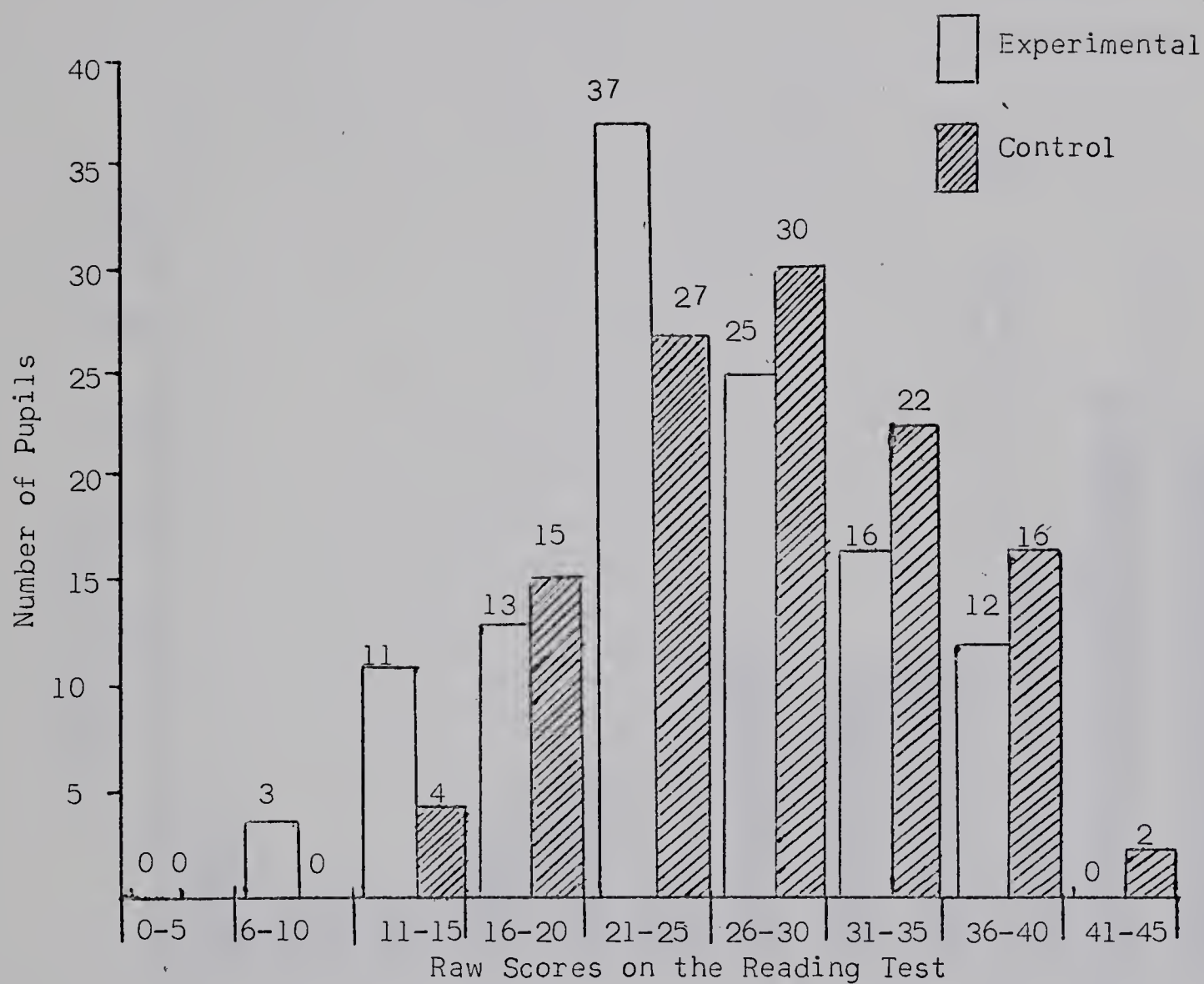


FIGURE 3

METROPOLITAN READING TEST RAW SCORES
FOR THE EXPERIMENTAL AND
CONTROL GROUPS

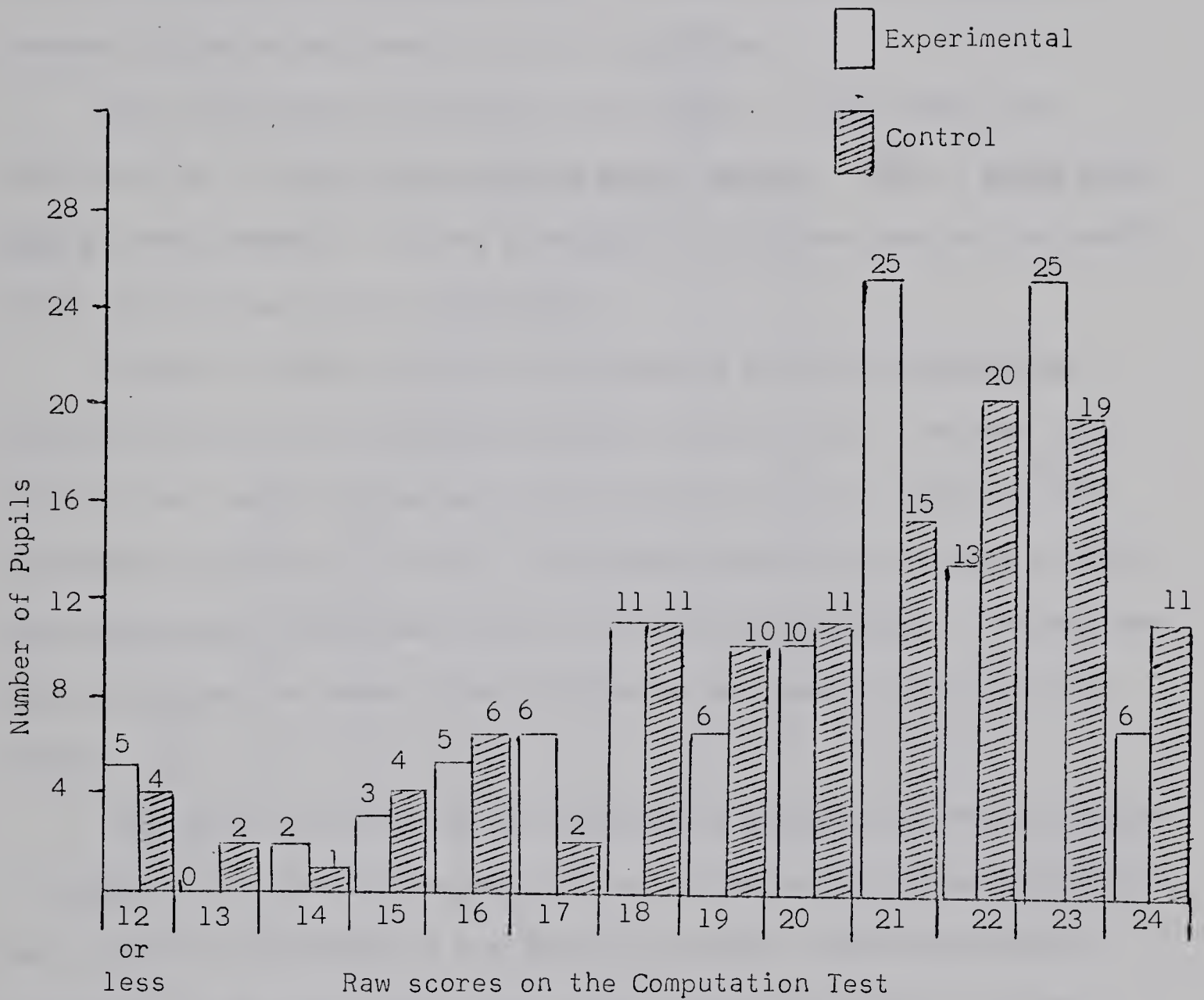


FIGURE 4

RAW SCORES ON THE COMPUTATION TEST FOR THE EXPERIMENTAL AND CONTROL GROUPS

pupil in the control group had less than twelve of the computations correct on the test. The experimental group had a mean of 20.02 and a standard deviation of 3.46. For the control group the mean was 20.10 and the standard deviation was 3.40. Using a t-test, the difference between the means was found to be not significant.

The intelligence quotients of the pupils in each group, as determined by the Otis Quick-Scoring Mental Ability Test: Alpha Test, Form A, were compared. Figure 5 contains a histogram showing the distribution of I.Q. scores for each group.

Figure 5 shows that the I.Q. scores of both the experimental group and the control group were normally distributed. The mean I.Q. score of the control group was 110.31 and the mean I.Q. score of the experimental group was 112.49. The standard deviations for the control and experimental groups were 11.05 and 13.27 respectively. A t-test was done to compare the means. The difference was shown to be not significant.

The pupils' ability to write equations representing the structure of additive and subtractive problems was estimated before the treatments were given. A histogram of the number of correct equations written by pupils in the experimental and control groups is shown in Figure 6.

Figure 6 shows that the ability to write equations for additive and subtractive problems was approximately the same for both groups. The mean raw score was 14.51 for the control group and 14.85 for the experimental group. The standard deviations of the scores for the control and experimental groups were 5.69 and 5.21 respectively. A t-test was

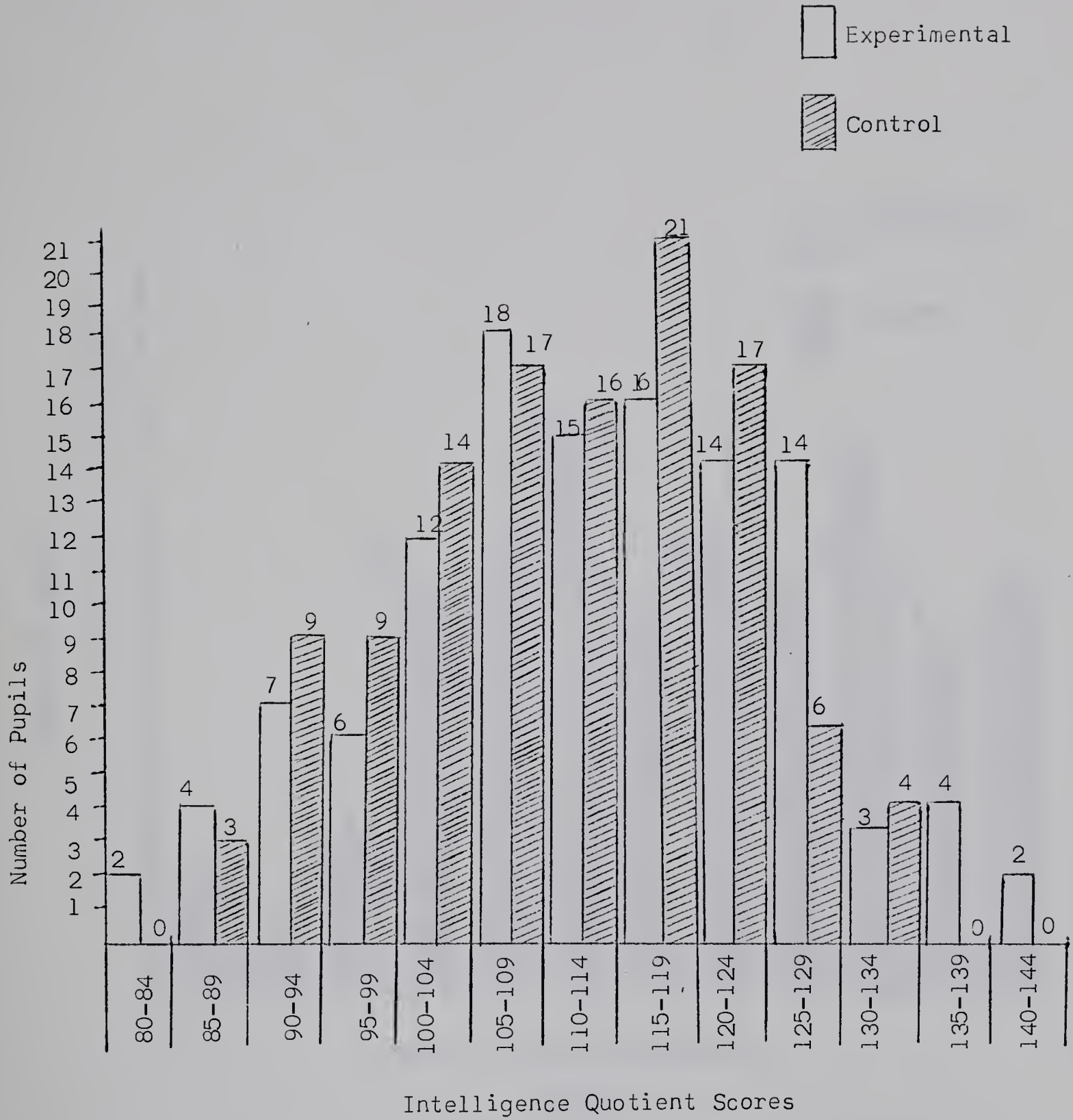


FIGURE 5

OTIS INTELLIGENCE QUOTIENT SCORES FOR THE EXPERIMENTAL AND CONTROL GROUPS

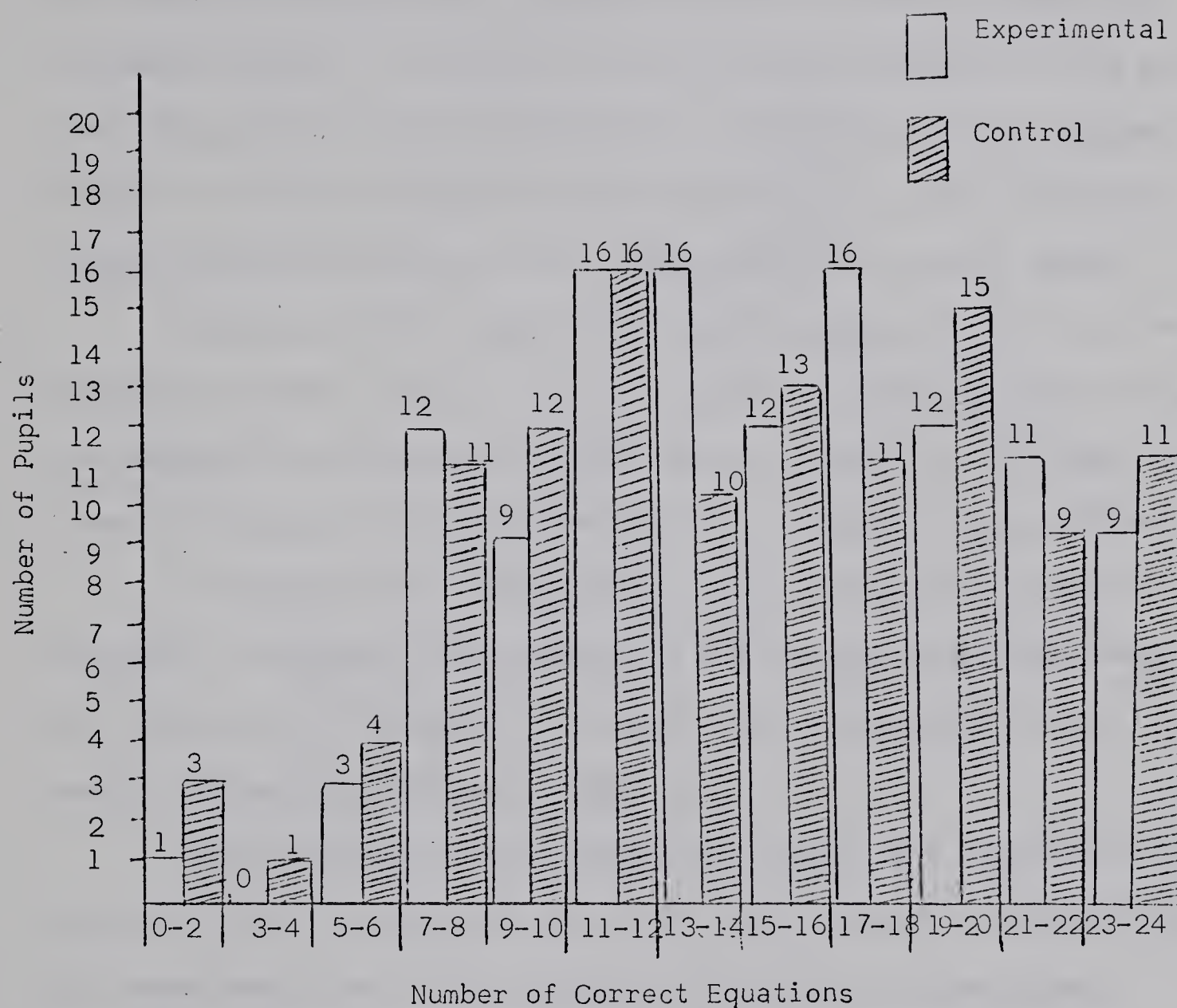


FIGURE 6

NUMBER OF CORRECT EQUATIONS ON THE PRETEST BY THE EXPERIMENTAL
AND CONTROL GROUPS

done to compare the means. The difference was shown to be not significant.

The ability of the pupils to arrive at correct numerical answers for additive and subtractive problems was also estimated before the treatments began. On the basis of the raw scores obtained on the pretest the pupils in the experimental and control groups were compared. Figure 7 contains histograms showing the number of correct numerical answers achieved by pupils in the experimental and control groups.

The mean for the scores by the control group was 13.61 and the standard deviation was 5.78. For the scores by the experimental group the mean was 14.22 and the standard deviation was 4.34. The means were compared using the t-test and found to be not significantly different.

In Figures 3 to 7 the scores of various tests were compared to determine how similar the experimental and control groups were before the treatments. A summary of the means and standard deviations of the pretest scores is presented in Table IX.

Only the means for the reading test scores were significantly different with the pupils in the control group having the higher scores. The experimental and control groups were similar in intelligence, ability to compute, and ability to solve verbal arithmetic problems.

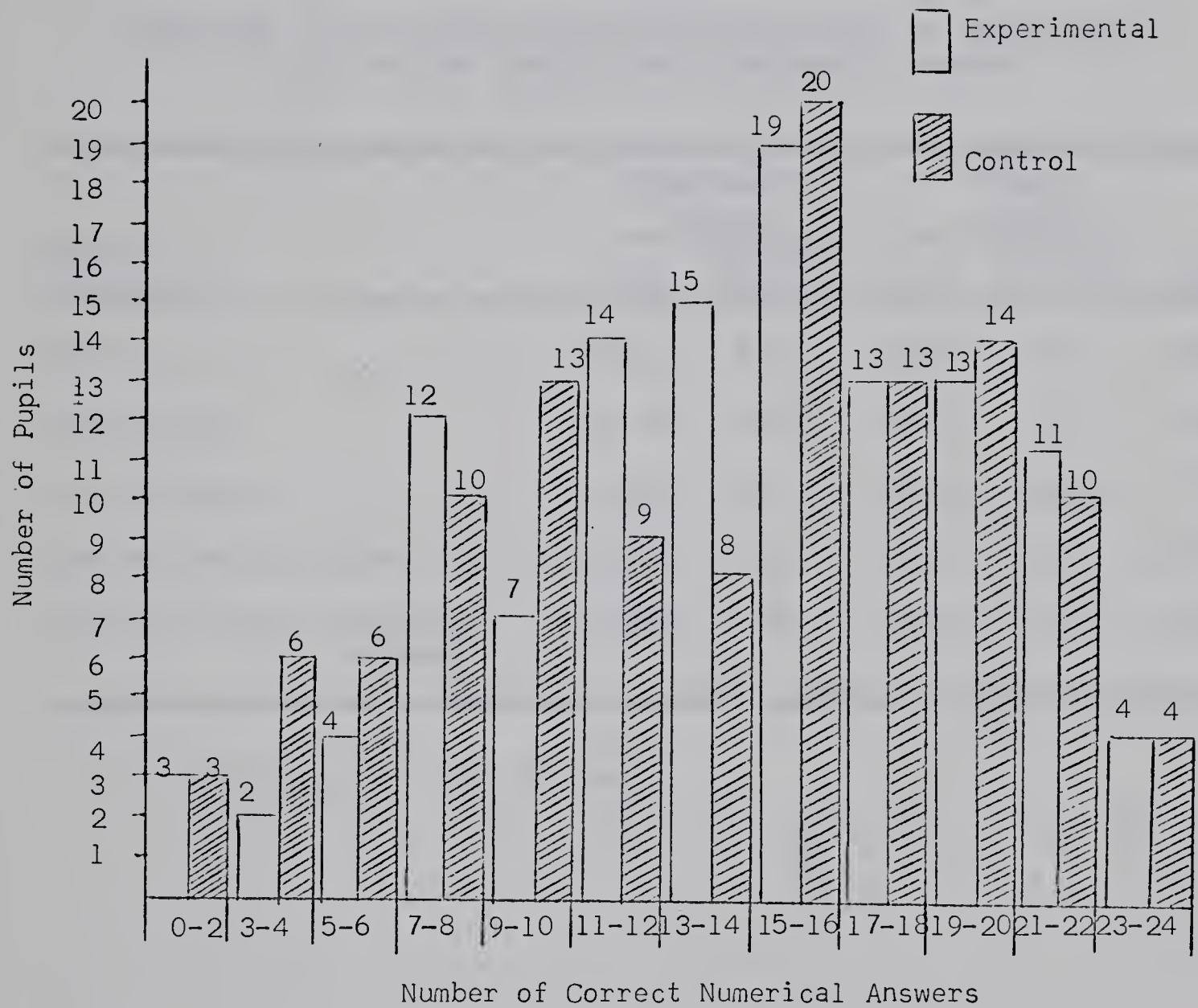


FIGURE 7

NUMBER OF CORRECT NUMERICAL ANSWERS ON THE PRETEST BY THE
EXPERIMENTAL AND CONTROL GROUPS

TABLE IX

COMPARISON OF THE MEANS AND STANDARD DEVIATIONS OF PRETREATMENT TESTS FOR THE CONTROL AND EXPERIMENTAL GROUPS

Aspect of Comparison	Experimental Group		Control Group		t-ratio
	Mean	Standard Deviation	Mean	Standard Deviation	
Reading	25.11	7.09	27.38	6.70	6.28 ¹
Computation	20.02	3.46	20.10	3.40	0.04
Intelligence	112.49	13.27	110.31	11.05	1.83
Problem solving (equations)	14.85	5.21	14.51	5.69	0.22
Problem solving (numerical answers)	14.22	4.34	13.61	5.78	0.70

¹ Significant at the .05 level.

II. ANALYSIS OF THE DATA CONCERNING THE HYPOTHESES

Evidence reported by various authors suggests that the groups in this study may be considered sufficiently similar to allow for the use of analysis of covariance to test the null hypotheses. Campbell and Stanley (1967, pp. 47-50) state that if the experimental and control groups are similar in their recruitment and if the scores on the pre-treatment data confirm the group similarity then this type of study approaches true experimentation. They recommend analysis of covariance for testing the null hypotheses if the similarity of the groups can be demonstrated.

The recruitment of the classes for this study was carefully described in Chapter III. An effort was made to select classes that were as similar as possible. Preliminary discussions with the teachers suggested that the characteristics of the classes in each group were similar.

Comparisons of the ages, arithmetic experiences outside the school, socio-economic status and rate of promotion of the pupils in each group further demonstrated the similarity of the groups. When the pretreatment scores were compared, only the means of the reading comprehension scores were found to be significantly different for the two groups. The effects of this difference were accounted for, however, by using the reading scores as a covariate in the analysis of covariance.

For the purposes of this study it was assumed that the experimental group and the control group were sufficiently similar in recruitment and pretreatment scores to allow for meaningful statistical

inference. On this assumption four null hypotheses were tested using analysis of covariance and the F-ratio test of significance.

Each of the four null hypotheses was tested for eight different subgroups of the total sample. Each subgroup was considered separately to determine if there was any sector of the school population for which the use of manipulative materials was particularly effective in the teaching of arithmetic problem solving. Table X shows the number of pupils in each of the subgroups considered in this study.

TABLE X
NUMBER OF PUPILS IN EACH SUBGROUP

Subgroup	Experimental	Control	Total
All pupils	117	116	233
Girls	54	54	108
Boys	63	62	125
High I.Q.	37	26	63
Average I.Q.	48	54	102
Low I.Q.	32	36	68
High S.E.S.	70	61	131
Low S.E.S.	47	55	102

The statistical analysis of the findings from the study is described below. Each null hypothesis is stated and the results are discussed.

Hypothesis 1. Pupils using manipulative materials during problem solving instruction do not develop significantly more ability to write equations representing the structure of additive and subtractive arithmetic problems than do pupils who use only a discussion method of instruction.

The average gain in problem solving ability from pretest to post test was compared for the two groups. Any differences in gain by the experimental group and the control group can be attributed to the use of manipulative materials by the experimental group. A summary of the average gain in ability to write equations for pupils in each subgroup is shown in Table XI.

TABLE XI
AVERAGE GAIN IN ABILITY TO WRITE CORRECT EQUATIONS

Subgroup	Experimental	Control
All pupils	4.92	5.66
Girls	4.63	5.31
Boys	5.03	6.16
High I.Q.	4.38	4.50
Average I.Q.	5.27	6.13
Low I.Q.	5.00	6.11
High S.E.S.	4.67	5.54
Low S.E.S.	5.30	6.00

The data in Table XI suggest that the pupils who used only a discussion method had an average gain in ability to write equations that was as great or greater than that of the pupils who used manipulative materials.

The null hypothesis was tested for each of the subgroups. The findings are shown in Table XII. The pupils' scores representing ability to write equations on the post test were used as the criterion scores. Scores representing reading comprehension, ability to compute and pretreatment ability to write equations were used as covariates. Each null hypothesis was tested at the .05 level of significance.

TABLE XII
STATISTICAL ANALYSIS OF IMPROVEMENT IN ABILITY TO WRITE
CORRECT EQUATIONS

Subgroup	F-ratio	Degrees of freedom		Test of null hypothesis at .05 level
		num.	den.	
All pupils	.16	1	229	not rejected
Girls	.03	1	104	not rejected
Boys	.08	1	121	not rejected
High I.Q.	.71	1	59	not rejected
Average I.Q.	.00	1	98	not rejected
Low I.Q.	.21	1	64	not rejected
High S.E.S.	.26	1	127	not rejected
Low S.E.S.	.00	1	98	not rejected

A study of Table XII reveals that the null hypothesis was not rejected for any of the subgroups being considered. The use of manipulative materials by the experimental group appeared to have no significant effect on the development of the pupils' ability to write correct equations for additive and subtractive problems.

Hypothesis 2. Pupils using manipulative materials during problem solving instruction do not develop significantly more ability to arrive at correct numerical answers for additive and subtractive problems than do pupils who use a discussion method of instruction.

Considering only the number of correct numerical answers, the Problem Solving Tests administered before and after the treatments were scored. The average gain by the pupils in the control group and the experimental group was compared to determine which method of instruction resulted in the most improvement. The mean gain by pupils in each group for various sectors of the school population is summarized in Table XIII.

A study of Table XIII reveals that the average gain in ability to arrive at correct numerical answers was about the same for pupils in the experimental and control groups when all pupils, girls, boys, pupils of high intelligence and pupils of high and low socio-economic status were considered separately. For pupils of low intelligence the control group gained an average of about two correct numerical answers more than did the experimental group. For pupils of average intelligence the experimental group gained an average of about two more correct numerical answers than did the control group.

TABLE XIII
AVERAGE GAIN IN ABILITY TO ARRIVE AT CORRECT NUMERICAL ANSWERS

Subgroup	Experimental	Control
All pupils	3.10	3.39
Girls	3.20	3.74
Boys	3.02	2.98
High I.Q.	2.59	2.38
Average I.Q.	4.02	2.87
Low I.Q.	2.34	4.72
High S.E.S.	3.41	3.02
Low S.E.S.	2.70	3.70

The null hypothesis that the use of manipulative materials made no significant difference in the amount of improvement in ability to arrive at correct numerical answers was tested. The findings for each subgroup of the school population are shown in Table XIV. For each subgroup the number of correct numerical answers on the post test was used as the criterion score. Scores representing ability to compute, the number of correct numerical answers on the pretest, and reading comprehension ability were used as covariates.

The information reported in Table XIV reveals that the null hypothesis was not rejected for any of the subgroups being considered. This information suggests that the use of manipulative materials did not have any significant effect on the improvement of the pupils' ability to

TABLE XIV

STATISTICAL ANALYSIS OF IMPROVEMENT IN ABILITY TO ARRIVE
AT CORRECT NUMERICAL ANSWERS

Subgroup	F-ratio	Degrees of freedom		Test of null hypothesis at .05 level
		num.	den.	
All pupils	1.35	1	229	not rejected
Girls	0.34	1	104	not rejected
Boys	1.05	1	121	not rejected
High I.Q.	0.10	1	59	not rejected
Average I.Q.	1.51	1	98	not rejected
Low I.Q.	0.23	1	64	not rejected
High S.E.S.	2.49	1	127	not rejected
Low S.E.S.	0.01	1	98	not rejected

solve verbal arithmetic problems.

The pupils in this study were tested to determine how much ability to solve verbal problems was retained by each group. The retention score of each pupil was determined by calculating the difference between the score on the pretest and the score on the same test administered about one month after the treatments. The following two null hypotheses concerning the retention of problem solving ability were tested.

Hypothesis 3. Pupils using manipulative materials during problem solving instruction do not retain significantly more ability to write

equations representing the structure of additive and subtractive problems than do pupils who use only a discussion method of instruction.

Scores representing the average amount of retention of ability to write correct equations for additive and subtractive problems are reported in Table XV.

TABLE XV
AVERAGE RETENTION SCORES OF ABILITY TO WRITE
EQUATIONS

Subgroup	Experimental	Control
All pupils	4.41	4.97
Girls	4.33	5.48
Boys	4.46	4.68
High I.Q.	3.38	3.96
Average I.Q.	5.56	5.31
Low I.Q.	4.44	5.50
High S.E.S.	4.27	4.93
Low S.E.S.	4.64	5.22

An analysis of Table XV shows that the pupils in the control group who used a discussion method had higher retention scores than did the pupils who used manipulative materials. This was true for every sector of the sample except pupils of average intelligence. There was, however, a gain by both groups of three or more in the number of correct equations between the pretest and the retention test. This gain

would suggest that both the treatment methods were effective in helping pupils retain the ability to write correct equations for additive and subtractive problems.

To test the null hypothesis that the use of manipulative materials had no effect on the amount of retention of ability to write correct equations, the scores of the test administered about a month after the treatments were used as the criterion scores. The scores from the reading test, the computation test, and the problem solving pre-test were used as the covariates. The findings for each subgroup of the sample are reported in Table XVI.

TABLE XVI
STATISTICAL ANALYSIS OF RETENTION OF ABILITY TO
WRITE CORRECT EQUATIONS

Subgroup	F-Ratio	Degrees of freedom		Test of null hypothesis at .05 level
		num.	den.	
All pupils	0.03	1	229	not rejected
Girls	1.92	1	104	not rejected
Boys	0.36	1	121	not rejected
High I.Q.	0.54	1	59	not rejected
Average I.Q.	0.63	1	98	not rejected
Low I.Q.	1.27	1	64	not rejected
High S.E.S.	0.11	1	127	not rejected
Low S.E.S.	0.01	1	98	not rejected

A study of the information contained in Table XVI shows that the use of manipulative materials by the experimental group had no significant effect on the retention of the ability to write correct equations for additive and subtractive problems. The null hypothesis was not rejected for any of the subgroups of the sample.

Hypothesis 4. Pupils using manipulative materials during problem solving instruction do not retain significantly more ability to arrive at correct numerical answers for additive and subtractive problems than do pupils who use only a discussion method of instruction.

Scores representing retention of ability to arrive at correct numerical answers were determined. These scores were determined by calculating the difference between the number of correct answers for the same problem solving test when it was administered before the treatments began and again about one month after the treatments. The average retention for the pupils in each of the subgroups is shown in Table XVII.

By studying Table XVII it can be seen that the pupils who used only the discussion method retained as much or more ability to arrive at correct answers than did the pupils who used manipulative materials. This was true for every subgroup of the sample except for boys, and for pupils of average intelligence. For these two subgroups pupils who used manipulative materials had the higher average.

The scores of the problem solving test administered about one month after the treatments were used as criterion scores to test the null hypothesis that there was no greater retention of ability to arrive at correct numerical answers by the pupils who used the manipulative

TABLE XVII
AVERAGE RETENTION SCORES OF ABILITY TO ARRIVE AT
CORRECT NUMERICAL ANSWERS

Subgroup	Experimental	Control
All pupils	3.29	3.59
Girls	2.76	4.11
Boys	3.76	2.98
High I.Q.	2.38	2.31
Average I.Q.	4.76	3.33
Low I.Q.	2.28	4.63
High S.E.S.	3.43	3.48
Low S.E.S.	3.11	3.55

materials. Differences in reading comprehension, ability to compute, and pretreatment ability to solve verbal problems were accounted for by using the scores from the appropriate tests as covariates. The findings for each subgroup of the sample are reported in Table XVIII.

A study of Table XVIII reveals that the null hypothesis was not rejected for any subgroup of the sample except for boys. Boys who used manipulative materials were able to retain significantly more ability to arrive at correct numerical answers than did the boys who used only the discussion method of instruction.

TABLE XVIII

STATISTICAL ANALYSIS OF RETENTION OF ABILITY TO ARRIVE AT
CORRECT NUMERICAL ANSWERS

Subgroup	F-ratio	Degrees of freedom		Test of null hypothesis at .05 level
		num.	den.	
All pupils	0.77	1	229	not rejected
Girls	1.28	1	104	not rejected
Boys	4.19	1	121	rejected
High I.Q.	0.03	1	59	not rejected
Average I.Q.	2.19	1	98	not rejected
Low I.Q.	0.01	1	64	not rejected
High S.E.S.	0.36	1	127	not rejected
Low S.E.S.	0.27	1	98	not rejected

V. SUMMARY OF FINDINGS

The findings of the study can be summarized as follows:

1. On the basis of the scores from the pretests, the experimental and control groups were sufficiently similar to allow for meaningful statistical analysis of the hypotheses. Only the means on the Metropolitan Reading Test differed significantly for the two groups with the pupils in the control group having the higher scores. However, this difference was controlled by using analysis of covariance to test the hypotheses.

2. The gain in ability to write correct equations was not

significantly different for the two methods of instruction. When due regard was given to differences in the pupils' ability to write equations before the treatments, to compute accurately, and to comprehend written material neither method resulted in significantly more gain.

3. The gain in ability to arrive at correct numerical answers was not significantly different for the two methods of instruction. When due regard was given to initial differences in ability to solve verbal problems, comprehend written materials, and compute accurately, neither method resulted in significantly more improvement.

4. The differences between the experimental group and the control group were not significant when retention of ability to write correct equations for additive and subtractive problems was considered. This was true for all subgroups of the sample when initial differences in ability to write equations, to compute accurately, and to comprehend written materials were accounted for.

5. When the effects of differences in initial ability to solve problems, compute accurately, and comprehend written materials were accounted for, boys who had used manipulative materials in their instruction retained significantly more ability to arrive at correct numerical answers than did boys who experienced only a discussion method of instruction. There was no significant difference between the methods for any other subgroup of the sample.

CHAPTER V

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

I. SUMMARY

The purpose of this study was to investigate the effectiveness of instruction which emphasized manipulative materials in developing ability to solve additive and subtractive verbal arithmetic problems.

Eight classes of grade three pupils participated in the study. The classes were selected on the basis of class size, achievement level, and type of arithmetic experiences previous to the study. Four of the classes were randomly assigned to the experimental group and given an eight lesson treatment which emphasized the use of manipulative materials to teach problem solving. The other four classes formed the control group. The treatment for the pupils in the control group was the same as that of the experimental group in every detail except that the control group used no manipulative materials.

On the basis of data collected during the study, the socio-economic status, age, intelligence quotients, reading comprehension ability, computational ability, and pretest problem solving ability of the pupils in the experimental group and the control group were compared. There were no significant differences in the characteristics of the two groups in socio-economic status, age, intelligence quotients, computational ability, and pretreatment problem solving ability. However, the mean of the control group on the Metropolitan Reading Test was

significantly greater than the mean of the experimental group on this test. To account for the differences in reading ability an analysis of covariance procedure was used. From the comparisons of the experimental and control groups it was concluded that they were sufficiently similar to allow for meaningful statistical analysis.

The improvement in problem solving ability that could be attributed to each of the treatment methods was estimated by administering the same problem solving test to each group before and after the treatment. Scores representing improvement by the experimental group and the control group were compared and analyzed to determine if the use of manipulative materials resulted in significantly more gain in ability to solve additive and subtractive problems. After a one-month interval the problem solving test was administered a third time to determine if the use of manipulative materials had any effect on the retention of ability to solve verbal arithmetic problems.

To determine if the use of manipulative materials was more effective in teaching problem solving for any specific sector of the school population, the results from various subgroups of the total sample were considered separately. Comparisons between the results for the experimental group and the control group were made for the following subgroups: all pupils, boys, girls, pupils of high intelligence, pupils of average intelligence, pupils of low intelligence, pupils of high socio-economic status, and pupils of low socio-economic status.

II. DISCUSSION OF THE FINDINGS

The Hypotheses

The results of the study in terms of the research hypotheses were as follows:

Hypothesis 1. The hypothesis that pupils who used manipulative materials during problem solving instruction would gain no more ability to write correct equations for additive and subtractive problems than would pupils using a discussion method of instruction was not rejected. There was no significant difference in the gain that could be attributed to either method of instruction when the post test scores were used as the criterion and the pretest scores were used as a covariate. This conclusion was the same for all the subgroups of the sample when the effect of differences in ability to comprehend written material was considered.

Hypothesis 2. The hypothesis that the use of manipulative materials would have no effect on the gain in development of ability to arrive at correct numerical answers for additive and subtractive problems was not rejected when the amount of gain was compared using post test scores as the criterion and pretest scores as a covariate. For each of the subgroups considered, there was no significant difference between the gain made by pupils who used manipulative materials and pupils who used only a discussion method of instruction.

Hypothesis 3. Retention of ability to write correct equations was

considered after a one-month interval. Since there was no significant difference between the retention by pupils who used the discussion method and retention by pupils who used manipulative materials in their instruction, the null hypothesis was not rejected for any subgroup of the sample.

Hypothesis 4. Boys who used manipulative materials during problem solving instruction retained significantly more ability to arrive at correct numerical answers for additive and subtractive problems than did the boys who had experienced only a discussion method of instruction. When the effect of differences in reading comprehension ability was accounted for, the null hypothesis was rejected only for boys.

There are several possible explanations for the differences in retention of ability to arrive at correct numerical answers by the boys. There is a possibility that the greater retention by the pupils in this subgroup reflects a necessity for an extended period of time for pupils to generalize from the concrete manipulations to the solution of verbal arithmetic problems. The treatment in this study involved only eight lessons and may thus have been too short for significant effects to appear in all of the subgroups.

It is also possible that the significant difference reflects a characteristic peculiar to boys. It is possible that boys have some common characteristic that results in greater retention of ability to arrive at correct numerical answers for verbal arithmetic problems when they are taught problem solving by having them manipulate concrete materials. Any attempt to describe the nature of this characteristic, if it exists, would be pure speculation in terms of this study and could

only be determined by further research.

When considering these possible explanations for the differences in retention among the boys, it should be noted that the use of manipulative materials was significantly more effective than the discussion method for boys only when retention of ability to arrive at correct numerical answers was being considered. There was no difference in the gain made by the boys in the experimental and control groups when initial ability to write correct equations or arrive at correct numerical answers was considered. The boys in the experimental and control groups did not differ either when retention of ability to write correct equations was considered. Only one of the thirty-two null hypotheses tested in this study was rejected. Thus, there is a possibility that the rejection of this null hypothesis for boys can be attributed to an error within the study itself.

Basically this study was designed to answer the following two questions: (1) Does the use of manipulative materials in problem solving instruction result in significantly more ability to solve additive and subtractive problems than does a method of instruction that stresses discussion? (2) Does the use of manipulative materials in problem solving instruction result in the retention of significantly more ability to solve additive and subtractive verbal arithmetic problems than does a method of instruction that stresses discussion?

The findings of this study led to the conclusion that the use of manipulative materials does not have a significant effect when improvement in ability to solve additive and subtractive problems is being

considered. When considering retention of ability to solve additive and subtractive problems, the findings of this study led to the conclusion that boys who have instruction using manipulative materials retain significantly more ability to arrive at correct numerical answers than do boys who experience a discussion method of instruction.

In interpreting the conclusions of this study, there are several limitations that should be kept in mind. There is the possibility that the eight lesson treatment period was too short to enable the pupils in the experimental group to generalize from the manipulation of the concrete materials to the solution of additive and subtractive verbal problems. Pupils who had not used manipulative materials before may have required much of the treatment period to become adjusted to their use. The manipulative materials may then have only confused the pupils, making problem solving as difficult for the experimental group as for the control group.

There is also the possibility that the findings were affected by the previous experiences of the pupils who participated in the study. To be entirely accurate in determining the effects of manipulative materials in learning problem solving, only pupils with no previous experiences involving manipulative materials should have participated in the study. An effort was made to select only classes that had not used concrete materials in their arithmetic lessons previous to this study. It is still probable, however, that all of the participating pupils had experiences which affected the results of this study. Any experiences in earlier grades, or at home, where the pupil used manipulative materials in a problem solving context, may have affected the

pupils' ability to do the problems in this study. It is thus possible that the pupils in each group reacted to the treatments in the same way because of similar previous experiences.

In any study such as this there is also the possibility that the teacher variable may have affected the findings. Although every effort was made to account for any differences in training and experience of the participating teachers, it was not possible to select teachers who were identical in their methods and abilities. The detailed directions supplied to the teachers were designed to reduce teacher differences. However, the possibility remains that what was done during the treatment lessons may have varied from class to class.

There is a possibility that manipulative materials could have been used more effectively to teach problem solving if they had been used in a different procedure. Harrison (1967, p. 74) suggested that the teacher should assist the internalization of the actions suggested in verbal arithmetic problem solving by having the pupils perform the actions with progressively less support from concrete materials. For example, the teacher could have had the pupils manipulate concrete objects until the actions suggested by the problems were understood. Next the pupils could have studied problems through pictorial representations of the actions. Then cognitive anticipations of the actions could have been suggested through discussion without any use of physical or pictorial representations. In this way the actions which were initially demonstrated with manipulative materials may have become internally understood by the pupils. Such a sequence of activities may

have been more effective than the sequence used in this study.

There is also the possibility that the pupils who participated in this study had already progressed past the stage of mental development when the manipulation of concrete materials would result in improvement in ability to solve verbal arithmetic problems. Adler (1966, p. 584) suggested such a possibility when he explained that what is 'concrete' to a person at one stage of development may not be considered as such for a person in another stage of development. For example, while the kindergarten child considers the union of two beads with four beads as a concrete operation but not the addition of two and four; the introductory algebra student considers two plus four as concrete but not $a + b$. If the pupils in this study had progressed past the stage of considering manipulative materials as 'concrete', the manipulation of these materials may be expected to result in few new insights and thus have little effect on the improvement of ability to solve verbal arithmetic problems. This possibility would suggest the need for further research at earlier grade levels.

III. CONCLUSIONS AND IMPLICATIONS

The results of this study give rise to a number of conclusions which have apparent educational implications:

1. The use of manipulative materials for a short period of time has no effect on development of ability to write correct equations for additive and subtractive problems.

2. The use of manipulative materials for a short period of time

has no effect on development of ability to arrive at correct numerical answers for additive and subtractive problems.

3. The use of manipulative materials for a short period of time has no effect on retention of ability to write equations for additive and subtractive problems.

4. A discussion method of instruction for a short period of time is as effective for developing ability to solve additive and subtractive problems as a method stressing the use of manipulative materials.

5. The use of manipulative materials for a short period of time is no more effective than a discussion method of instruction for teaching problem solving to girls, pupils of high intelligence, pupils of average intelligence, pupils of low intelligence, pupils of high socio-economic status, and pupils of low socio-economic status.

These conclusions imply that if manipulative materials are to be used effectively in teaching problem solving, they must be used for an extended period of time. A sudden emphasis on the use of manipulative materials appears to be of little value in developing problem solving ability.

6. There is some indication that the use of manipulative materials aids boys to retain ability to arrive at correct numerical answers for additive and subtractive problems.

This conclusion implies a need for the use of manipulative materials in teaching problem solving to boys. More research is needed, however, to determine the specific value of manipulative

materials for boys. The procedures by which manipulative materials could be used most effectively to teach problem solving also requires further investigation.

IV. SUGGESTIONS FOR FURTHER RESEARCH

This study has suggested several questions that should be explored by future experimentation. Some of these suggestions are presented below:

1. Would the findings be the same if the experiment reported here was replicated with tighter experimental controls and a longer treatment period?
2. Are manipulative materials effective for teaching other types of verbal arithmetic problems than those considered in this study?
3. What is the relationship between the level of cognitive development of the pupil and the effectiveness of instruction involving manipulative materials?
4. Are other instructional aids such as pictorial materials effective for the teaching of verbal arithmetic problem solving?
5. Can manipulative materials be effectively used to teach various other aspects of the arithmetic curriculum, such as the geometrical concepts of length, area and volume?
6. Can manipulative materials be used effectively to aid pupils in a discovery of mathematical concepts?
7. Is the use of manipulative materials particularly effective for teaching problem solving to boys, and if so, why?

8. What are the exact procedures by which manipulative materials can be used most effectively to teach various mathematical concepts?

9. What, if any, effect does the use of manipulative materials have on the thought processes of children?

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APPENDIX A

LESSON I

1. May had 3 oranges in a box. Her mother gave her 5 more oranges. How many oranges did May have then?

_____	$N - 5 = 3$	Computation
_____	$5 + 3 = N$	
_____	$N - 3 = 5$	
_____	$3 + 5 = N$	

May had _____ oranges after her mother gave her some more.

2. 6 toy boats were in a pond. Don put 3 more boats in the pond. Then there were how many boats in the pond?

_____	$N - 3 = 6$	Computation
_____	$3 + 6 = N$	
_____	$N - 6 = 3$	
_____	$6 + 3 = N$	

Then there were _____ boats in the pond.

3. Ellen had 12 flowers. She gave 5 of them to her friends. How many flowers did Ellen have left?

_____	$N + 5 = 12$	Computation
_____	$12 - 5 = N$	
_____	$12 - N = 5$	
_____	$5 + N = 12$	

Ellen had _____ flowers left after she gave 5 to her friends.

4. Bill had 14 books. He gave 9 of them to Tom to carry. How many books did Bill have left to carry?

_____	$14 - N = 9$	Computation
_____	$14 - 9 = N$	
_____	$N + 9 = 14$	
_____	$9 + N = 14$	

Bill had _____ books left to carry.

5. 23 birds were eating. 14 of them flew away. Then how many birds were eating?

_____	$23 - 14 = N$	Computation
_____	$23 - N = 14$	
_____	$N + 14 = 23$	
_____	$14 + N = 23$	

Then there were _____ birds eating.

6. 13 cars were parked in the parking lot. Then 18 more cars came to park. How many cars were parked in the lot then?

Arithmetic sentence

Computation

Then there were _____ cars parked in the lot.

7. There were 20 spools in a box. Rose took 7 spools out to play with them. Then how many spools were in the box?

Arithmetic sentence

Computation

Then there were _____ spools in the box.

8. There were 28 chocolate bars at the store. Some boys bought 15 of them. Then how many chocolate bars were left at the store?

Arithmetic sentence

Computation

There were _____ chocolate bars left at the store then.

9. John saw 9 birds in a tree. Then 11 more birds came to the tree. How many birds were in the tree then?

Arithmetic sentence

Computation

10. Betty painted 6 pictures, and June painted 7 pictures. The girls painted how many pictures in all?

Arithmetic sentence

Computation

The girls painted _____ pictures in all.

LESSON II

1. June had 6 leaves in her collection. Then Carol gave her some more. Now June has 24 leaves. How many leaves did Carol give to June?

_____	$K + 6 = 24$	Computation
_____	$6 + K = 24$	
_____	$24 - 6 = K$	
_____	$24 - K = 6$	

Carol gave June _____ leaves.

2. Ellen had 7 pennies in her bank. Mother gave her some more pennies to put in her bank. Then Ellen had 22 pennies in her bank. How many pennies did mother give her?

_____	$P + 7 = 22$	Computation
_____	$22 - P = 7$	
_____	$22 - 7 = P$	
_____	$7 + P = 22$	

Mother gave Ellen _____ pennies.

3. Jim had some baseball cards. Bill gave him 15 more baseball cards. Now Jim has 31 baseball cards. How many cards did Jim have to begin with?

_____	$31 - 15 = M$	Computation
_____	$15 + M = 31$	
_____	$M + 15 = 31$	
_____	$31 - M = 15$	

Jim had _____ baseball cards to begin with.

4. Bill was given 13 new stamps for his collection. Then he had 25 stamps in all. How many stamps did Bill have before he was given the 13 new stamps?

_____	$J + 13 = 25$	Computation
_____	$25 - 13 = J$	
_____	$13 + J = 25$	
_____	$25 - J = 13$	

Bill had _____ stamps before he was given the 13 new stamps.

5. Dick had 18 old coins last week. He bought some more old coins this week. Now he has 26 old coins. He bought how many old coins this week?

_____	$26 - 18 = U$	Computation
_____	$18 + U = 26$	
_____	$26 - U = 18$	
_____	$U + 18 = 26$	

Dick bought _____ old coins this week.

6. After Jim made 4 new model airplanes he had 16 in all. How many model airplanes did Jim have before he made some more?

Arithmetic sentence

Computation

Jim had _____ model airplanes before he made some more.

7. Sue bought a book of 12 paper dolls. Then she had 31 paper dolls. Sue had how many paper dolls at first?

Arithmetic sentence

Computation

Sue had _____ paper dolls before she bought some more.

8. Carol had 17 records. Her aunt gave her some more records. Then Carol had 30 records. How many records did Carol's aunt give her?

Arithmetic sentence

Computation

Carol's aunt gave Carol _____ records.

9. At the toy store Anne counted 23 china dolls on a table. A clerk put more china dolls with them. Then Anne counted 31 china dolls. How many china dolls did the clerk put on the table?

Arithmetic sentence

Computation

The clerk put _____ more china dolls on the table.

10. Don had some marbles in a bag. At school he won 15 more. Then Don had 28 marbles. How many marbles did Don have before he won some more?

Arithmetic sentence

Computation

Don had _____ marbles before he won some more.

LESSON III

1. 26 boys took their bikes to the beach and 17 boys walked to the beach. How many boys were at the beach all together?

Arithmetic sentence

Computation

There were _____ boys at the beach all together.

2. Dick had 43 shells. He gave away 19 of them. How many shells did Dick have left?

Arithmetic sentence

Computation

Dick had _____ shells left.

3. Ann made some cookies one day. The next day she made 20 more cookies. Then she had 40 cookies in all. How many cookies did Ann make the first day?

Arithmetic sentence

Computation

Ann made _____ cookies the first day.

4. Tom had 18 pencils. His father gave him some more pencils. Then Tom had 30 pencils in all. How many pencils did Tom's father give him?

Arithmetic sentence

Computation

Tom's father gave Tom _____ pencils.

5. There were 32 pupils in Joan's class. At noon 13 went home for lunch. How many children in Joan's class did not go home for lunch?

Arithmetic sentence

Computation

_____ children in Joan's class did not go home for lunch.

6. Nancy found 11 empty bottles on the street. Then she had 24 bottles. How many bottles did Nancy have before she found 11 more?

Arithmetic sentence

Computation

Nancy had _____ bottles before she found 11 more.

7. Bill have 14 cents and Tom had 23 cents. How much money did the boys have together?

Arithmetic sentence

Computation

The boys had _____ cents together.

8. Tom bought 7 small prizes for a party. Roy gave Tom some more prizes. Then Tom had 15 prizes for the party. How many prizes did Roy give to Tom?

Arithmetic sentence

Computation

Roy gave Tom _____ prizes for the party.

9. Tom blew up 16 balloons before school. After school he blew up some more balloons. Altogether he had blown up 25 balloons. How many balloons did Tom blow up after school?

Arithmetic sentence

Computation

Tom blew up _____ balloons after school.

10. Joey had 28 cents. He spent 12 cents on candy. How much money did Joey have left after he bought candy?

Arithmetic sentence

Computation

Joey had _____ cents left after he bought some candy.

LESSON IV

1. Tom had some pictures of dogs. He gave 8 of them to his sister. Then Tom had 7 pictures left. How many pictures of dogs did Tom have before he gave his sister some?

$$\begin{array}{l} \underline{\quad\quad} 8 - n = 7 \\ \underline{\quad\quad} n - 8 = 7 \\ \underline{\quad\quad} 8 + 7 = n \\ \underline{\quad\quad} n - 7 = 8 \end{array}$$

Computation

Tom had pictures of dogs before he gave his sister some.

2. Nancy picked some flowers. She gave 13 of them to a friend. Then she had 13 flowers left. How many flowers did Nancy pick?

$$\begin{array}{l} \underline{\quad\quad} 13 + 13 = H \\ \underline{\quad\quad} H + 13 = 13 \\ \underline{\quad\quad} H - 13 = 13 \\ \underline{\quad\quad} 13 - H = 13 \end{array}$$

Computation

Nancy picked flowers in all.

3. Jack had 33 cents to spend. He bought a toy horse. Then he had 10 cents left. How much money did Jack spend for the toy horse?

$$\begin{array}{l} \underline{\quad\quad} 33 - n = 10 \\ \underline{\quad\quad} 33 - 10 = n \\ \underline{\quad\quad} n + 10 = 33 \\ \underline{\quad\quad} 10 + n = 33 \end{array}$$

Computation

Jack spent cents for the toy horse.

4. Betty's mother made 24 cupcakes. At supper her family ate some of the cupcakes. Then there were 9 cupcakes left. How many cupcakes did the family eat at supper?

$$\begin{array}{l} \underline{\quad\quad} 24 - 9 = M \\ \underline{\quad\quad} 9 + M = 24 \\ \underline{\quad\quad} M + 9 = 24 \\ \underline{\quad\quad} 24 - M = 9 \end{array}$$

Computation

The family ate cupcakes at supper.

5. Carol had some coins. She lost 16 of her coins. Then she had 18 coins left. How many coins did Carol have before she lost some?

Arithmetic sentence

Computation

Carol had coins before she lost some of them.

6. Don caught 32 butterflies. Some of the butterflies got away. Then Don only had 19 butterflies left. How many butterflies got away?

Arithmetic sentence

Computation

_____ butterflies got away.

7. A farmer had 17 rabbits in a pen. Some of the rabbits ran away. Then the farmer had only 5 rabbits left. How many rabbits ran away?

Arithmetic sentence

Computation

_____ rabbits ran away.

8. Mr. Jones planted some trees in his yard. After 9 of them died, there were only 21 trees left. How many trees did Mr. Jones plant in all?

Arithmetic sentence

Computation

Mr. Jones planted _____ trees in all.

9. There were 24 cookies on a tray. Some girls ate some of the cookies. Then there were only 8 cookies left. How many cookies did the girls eat?

Arithmetic sentence

Computation

The girls ate _____ cookies.

10. Bill had some empty bottles in his basement. After he sold 24 bottles he had 13 bottles left. How many bottles did Bill have before he sold some?

Arithmetic sentence

Computation

Bill had _____ bottles before he sold some of them.

LESSON V

1. The third-grade boys had 16 marbles. After they won some more from another class they had 32 in all. How many more marbles did they win from the other class?

Arithmetic sentence

Computation

The boys won _____ marbles from the other class.

2. Ann found 43 nuts in the woods. She took some of the nuts to school. Then she had 15 left. How many nuts did Ann take to school?

Arithmetic sentence

Computation

Ann took _____ nuts to school.

3. Mother had a box of ice cream cones. After she used 16 of the cones there were 15 cones left. How many cones were in the box before Mother used some?

Arithmetic sentence

Computation

There were _____ cones in the box before Mother used some.

4. There were some books on the shelf in Joan's room. After her mother put 6 more books on the shelf Joan counted 21 books. How many books were on the shelf before Joan's mother put some more on?

Arithmetic sentence

Computation

There were _____ books on the shelf before Joan's mother put some more on.

5. 9 boys were playing in the playground. After some more boys came there were 20 boys in the playground. How many more boys came?

Arithmetic sentence

Computation

_____ more boys came.

6. After Sally got 11 points in a dart game she had 30 points in all. How many points did Sally have before she made the 11 more?

Arithmetic sentence

Computation

Sally had _____ points before she made the 11 more.

7. June had 7 toy bears. She got some more for her birthday. Now she has 19 bears. How many toy bears did she get for her birthday?

Arithmetic sentence

Computation

June got _____ toy bears for her birthday.

8. Miss Brown had 37 pieces of paper for an art lesson. She gave each child **one sheet** of paper. Then she had 4 sheets left. How many sheets of paper did she give to the children?

Arithmetic sentence

Computation

Miss Brown gave the children _____ sheets of paper.

9. Mary had 20 stamps in her collection. Her brother gave her some more stamps. Then Mary had 34 stamps in all. How many stamps did Mary get from her brother?

Arithmetic sentence

Computation

Mary got _____ stamps from her brother.

10. Bill had some snails. He sold 8 snails to Tom. Then Bill had 14 snails left. How many snails did Bill have before he gave some to Tom?

Arithmetic sentence

Computation

Bill had _____ snails before he gave some to Tom.

LESSON VI

1. Mary had some pine cones. She gave Carol 22 cones to take home. Then Mary had 13 cones left. How many cones did Mary have before she gave Carol some?

Arithmetic sentence

Computation

Mary had _____ cones before she gave Carol some.

2. Some children were playing a dart game. Then 7 more children came to play. Now there were 12 children playing the dart game. How many children were playing the dart game before the 7 more children came to play?

Arithmetic sentence

Computation

There were _____ children playing the dart game before the 7 more children came to play.

3. 26 girls were having a party. After some of the girls went home, there were only 11 girls left at the party. How many girls went home?

Arithmetic sentence

Computation

_____ girls went home.

4. After Joe got 10 cents from his grandmother he had 24 cents in all. How much money did Joe have before his grandmother gave him some more?

Arithmetic sentence

Computation

Joe had _____ cents before his grandmother gave him some more.

5. The class drew 28 pictures of animals. They put 16 of the pictures in a book. How many pictures were left over?

Arithmetic sentence

Computation

_____ pictures were left over.

6. Betty had a box of crayons. After she gave 6 crayons to Jane she had 18 crayons left. How many crayons did Betty have to begin with?

Arithmetic sentence

Computation

Betty had _____ crayons to begin with.

7. There were some pictures on the bulletin board. The teacher put up 16 more pictures. Then there were 29 pictures on the bulletin board in all. How many pictures were up before the teacher put 16 more pictures on the bulletin board?

Arithmetic sentence

Computation

There were _____ pictures on the bulletin board before the teacher put some more up.

8. Tom read 21 pages in a book about dogs before lunch. After lunch he read some more. Then he had read 34 pages in all. How many pages did Tom read after lunch?

Arithmetic sentence

Computation

Tom read _____ pages after lunch.

9. Peggy made some cookies. She took 20 of the cookies to a party. Then she had 15 cookies left. How many cookies did Peggy make in all?

Arithmetic sentence

Computation

Peggy made _____ cookies in all.

10. Bill found 42 shells. He gave some of them to Tom. Then Bill had 27 shells left. How many shells did Bill give to Tom?

Arithmetic sentence

Computation

Bill gave Tom _____ shells.

LESSON VII

1. After a farmer bought 18 pigs he had 36 pigs in all. How many pigs did the farmer have before he bought the 18 more?

Arithmetic sentence

Computation

The farmer had _____ pigs before he bought the 18 more.

2. Sam had 36 marbles. At recess he won 9 more marbles. How many marbles did Sam have then?

Arithmetic sentence

Computation

Sam had _____ marbles then.

3. Mary had some cookies. At the store she bought 24 more cookies. Then Mary had 36 cookies in all. How many cookies did Mary have before she bought some more?

Arithmetic sentence

Computation

Mary had _____ cookies before she bought some more.

4. Sam had 36 marbles. At recess he lost 9 marbles. How many marbles did Sam have then?

Arithmetic sentence

Computation

Sam had _____ marbles then.

5. 8 children were playing in the park. 11 more children came to play. How many children were playing in the park then?

Arithmetic sentence

Computation

There were _____ children playing in the park then.

6. There were some jars of blue paint on a shelf in a store. The clerk sold 18 jars. Then there were 15 jars left on the shelf. How many jars of blue paint were on the shelf before the clerk sold the 18 jars?

Arithmetic sentence

Computation

There were _____ jars of blue paint on the shelf before the clerk sold the 18 jars.

7. A clerk said, "I sold 28 pencils today. Now I have 12 pencils left." How many pencils did the clerk have to begin with?

Arithmetic sentence

Computation

The clerk had _____ pencils to begin with.

8. Ellen had 22 pieces of colored chalk. She bought some more and then she had 30 pieces of chalk. How many pieces did she buy?

Arithmetic sentence

Computation

Ellen bought _____ pieces of chalk.

9. A clerk had 40 large sheets of red paper to sell. After he had sold some of them, he found he had 17 sheets left. How many sheets had he sold?

Arithmetic sentence

Computation

The clerk had sold _____ sheets.

10. Dick said, "After 16 people get off the bus, there will be 15 left on the bus." How many people were on the bus?

Arithmetic sentence

Computation

There were _____ people on the bus.

LESSON VIII

1. Bobby had a score of seventeen points in the bean-bag game. He made 8 points on his next throw. How many points does Bobby have now?

Arithmetic sentence

Computation

Bobby has _____ points now.

2. A farmer had 14 cows. He bought some more cows. Then he had 30 cows in all. How many cows did the farmer buy?

Arithmetic sentence

Computation

The farmer bought _____ cows.

3. Seven girls went into an art store. Mary counted all the people and said, "There are 31 people in here now." How many people were in the store before the 7 girls went in?

Arithmetic sentence

Computation

There were _____ people in the store before the 7 girls went in.

4. Mrs. Young baked 24 cupcakes. Some of the cupcakes were eaten for lunch. Then there were 16 cupcakes left. How many cupcakes were eaten for lunch?

Arithmetic sentence

Computation

There were _____ cupcakes eaten for lunch.

5. 17 boys were playing ball. Nine of the children went home. Then how many boys were left to play ball?

Arithmetic sentence

Computation

There were _____ boys left to play ball.

6. Tom bought 38 apples. He gave 24 of them to the class for a party. How many apples did Tom have left?

Arithmetic sentence

Computation

Tom had _____ apples left.

7. 6 children were helping with an art fair. Some more children came to help. Then there were 14 children helping. How many more children came to help?

Arithmetic sentence

Computation

_____ more children came to help.

8. Twenty birds were eating on the ground. Then 14 more birds came to eat. How many birds were eating in all then?

Arithmetic sentence

Computation

There were _____ birds eating in all.

9. Joey had 29 toy cars. He gave some of the toy cars to his friends. Then Joey had 13 cars left. How many cars did Joey give to his friends?

Arithmetic sentence

Computation

Joey gave _____ toy cars to his friends.

10. Greg had 35 cents to spend. He bought a water gun for 19 cents. How much money did Greg have left?

Arithmetic sentence

Computation

Greg had _____ cents left.

APPENDIX B

LESSON 1

Discussion Method

All the problems in lesson one are of two types. The action suggested by each can be represented by arithmetic sentences of the form $4 + 8 = N$ or $8 - 4 = N$.

The procedure to follow in this problem-solving lesson is as follows:

- 1) Have the class read the first problem silently.
- 2) Have one pupil read the problem aloud.
- 3) Ask the pupils to explain what action is suggested by the problem.
Example: In problem 1, the action is a combining action.
May had 3 oranges. She combined the 5 oranges her mother gave her with the 3 she already had.
- 4) Direct the pupils to put an X in front of the arithmetic sentence that best expresses the action suggested by the problem.
- 5) Direct the pupils to compute to find the unknown term in the arithmetic sentence.
- 6) Ask one pupil to write the correct arithmetic sentence and the computation on the blackboard.
- 7) Direct the other pupils to verify their own work by comparing it with the one on the blackboard. If theirs is incorrect, ask them to change it. Try to have them understand why it is not correct.
- 8) Direct the pupils to complete the verbal sentence that answers the question asked by the problem. Then have one pupil read the sentence that would answer the question in the problem.
- 9) Ask the pupils if they understand each of the ideas covered in steps one to eight.
- 10) Repeat the same procedure for problems 2 to 5.
- 11) Use the same procedure in problems 6 to 10 except that in these problems the pupils should be directed to write their own arithmetic sentence to describe the action suggested by the problem.

LESSON 2

Discussion Method

There are two types of problems in lesson 2. The action suggested by each type of problem can be represented by arithmetic sentences of the form $N + 3 = 6$ or $3 + N = 6$.

The procedure to follow in this problem solving lesson is as follows:

- 1) Have the class read the first problem silently.
- 2) Have one pupil read the problem aloud.
- 3) Ask the pupils to explain what action is suggested by the problem.
Example: In problem 1, the action is a combining action. June had 6 leaves. She combined an unknown number of leaves with the 6 she already had. Then she had a total of 24 leaves.

In problem 3, the action is also a combining action. Jim had an unknown number of cards. He combined 15 more cards with those he already had. Then he had a total of 31 cards.
- 4) Direct the pupils to put an X in front of the arithmetic sentence that best expresses the action suggested by the problem.
- 5) Direct the pupils to compute to find the unknown term. Through discussion lead the pupils to understand that it is necessary to subtract to find the unknown term in arithmetic sentences of the form $N + 3 = 6$ or $3 + N = 6$.
- 6) Ask one pupil to write the correct arithmetic sentence and the computation on the blackboard.
- 7) Direct the pupils to verify their own work by comparing it with the one on the blackboard. If theirs is incorrect, ask them to change it. Try to have them understand why it is not correct.
- 8) Direct the pupils to complete the verbal sentence that answers the question asked by the problem. Then have one pupil read the sentence that would answer the question in the problem.
- 9) Ask the pupils if they understand each of the ideas covered in steps 1 to 8.
- 10) Repeat the same procedure for problems 2 to 5.
- 11) Use the same procedure in problems 6 to 10 except that in these problems the pupils should be directed to write their own arithmetic sentences to describe the action suggested by the problem.

LESSON 3

Discussion Method

Lesson 3 contains problems similar to those studied in lessons 1 and 2.

The procedure to follow in this problem solving lesson is as follows:

- 1) Have the class read the first problem silently.
- 2) Have one member of the class read the problem aloud.
- 3) Ask the pupils to explain what action is suggested by the problem.
It is important that the pupils be drawn to think about the action that would take place if the situation described by the problem were really taking place. It is the aim of these lessons to make the pupil understand that this suggested action can be represented by the arithmetic sentence.
- 4) Direct the pupils to write the arithmetic sentence that best expresses the action suggested by the problem.
- 5) Direct the pupils to compute to find the unknown term in the arithmetic sentence. It may be necessary to use discussion to review the computations used in lessons 1 and 2.
- 6) Ask one pupil to write the correct arithmetic sentence and the computation on the blackboard.
- 7) Direct the pupils to verify their own work by comparing it with the one on the blackboard.
- 8) Direct the pupils to complete the verbal sentence that answers the question asked by the problem. Then have one pupil read the sentence that would answer the question in the problem.
- 9) Repeat the same procedure for problems 2 to 10.

LESSON 4

Discussion Method

There are two types of problems in lesson 4. The action suggested by each type of problem can be represented by arithmetic sentences of the form $N - 3 = 6$, or $9 - N = 6$.

The procedure to follow in this problem solving lesson is as follows:

- 1) Have the class read the first problem silently.
- 2) Have one pupil read the problem aloud.
- 3) Ask the pupils to explain what action is suggested by the problem.
Example: In problem 1, the action is a separating action. Tom has a group of pictures. He separates the 8 he gives to his sister from the whole group of pictures. Then he has only 7 pictures left.

In problem 3, the action is also a separating action. Jack has a group of 33 cents. He separates a group of cents from the 33 he had to begin with. Then he has only 10 cents left.

- 4) Direct the pupils to put an X in front of the arithmetic sentence that best expresses the action suggested by the problem.
- 5) Direct the pupils to compute to find the unknown term in the arithmetic sentence. Through discussion, allow the pupils to explore the various possibilities until they decide that they must add to find the missing term for problems like the first one, and they must subtract to find the missing term for problems like the third problem.
- 6) Ask one pupil to write the correct arithmetic sentence and the computation on the blackboard.
- 7) Direct the pupils to verify their work by comparing it with the example on the blackboard.
- 8) Direct the pupils to complete the verbal sentence. Then have one pupil read the sentence that would answer the question to the problem.
- 9) Repeat the same procedure for problems 2 to 4.
- 10) Use the same procedure for problems 5 to 10 except that for these problems the pupils should be directed to write their own arithmetic sentences to describe the action suggested by the problem.

LESSONS 5 and 6

Discussion Method

Lessons 5 and 6 are a review of the types of problems studied in lessons 2 and 4.

The procedure to follow in these problem solving lessons is as follows:

- 1) Direct the pupils to do problems 1 to 5 on their own. Allow sufficient time for them to finish.
- 2) Then beginning with problem 1, direct a pupil to read the problem aloud.
- 3) Lead the pupils in a discussion of the action suggested by the problem.
- 4) Ask one pupil to write the arithmetic sentence and the appropriate computation on the blackboard.
- 5) Allow the pupils to discuss the work on the blackboard and direct them to verify their own work.
- 6) Ask a pupil to read the appropriate verbal sentence.
- 7) Use the same procedure for problems 6 to 10.

LESSONS 7 and 8

Discussion Method

Lessons 7 and 8 are a review of all six types of problems studied in the earlier lessons.

The procedure to use for lessons 7 and 8 is as follows:

- 1) Direct the pupils to do problems 1 to 10 on their own. Allow sufficient time for them to finish.
- 2) Then beginning with problem 1 direct a pupil to read the problem aloud.
- 3) Lead the pupils in a discussion of the action suggested by the problem.
- 4) Ask one pupil to write the arithmetic sentence and the appropriate computation on the blackboard.
- 5) Allow the pupils to discuss the work on the blackboard and direct them to verify their own work.
- 6) Ask a pupil to read the appropriate verbal sentence.
- 7) Do the same with each of the other problems.

LESSON 1

Concrete Materials Method

All of the problems in Lesson 1 are of two types. The action suggested by each can be represented by arithmetic sentences of the form $4 + 8 = N$ or $8 - 4 = N$.

The procedure to follow in this problem solving lesson is as follows:

- 1) Have the class read the first problem silently.
- 2) Have one pupil read the problem aloud.
- 3) Ask the pupils to explain what action is suggested by the problem.
Example: In problem 1, the action is a combining action. Mary had 3 oranges. She combined the 5 oranges her mother gave her with the 3 she already had.
- 4) Ask the pupils to use their concrete materials to show the action that is suggested by the problem.
Example: For problem 1, a pupil may place 3 objects in one pile and show the action of combining 5 more objects with the 3 already in the pile.
- 5) Ask one of the pupils to demonstrate the action for the class. Encourage discussion of the action being demonstrated.
- 6) Direct the pupils to put an X in front of the arithmetic sentence that best expresses the action suggested by the problem.
- 7) Direct the pupils to compute to find the unknown term in the arithmetic sentence.
- 8) Ask one pupil to write the correct arithmetic sentence and the computation on the blackboard.
- 9) Direct the pupils to verify their own work by comparing it with the one on the blackboard. If theirs is not correct, ask them to change it. Try to have them understand why it is not correct.
- 10) Direct the pupils to complete the verbal sentence that answers the question asked by the problem. Then have one pupil read the sentence that would answer the question in the problem.
- 11) Ask the pupils if they understand each of the ideas covered in steps 1 to 10.
- 12) Repeat the same procedure for problems 2 to 5.
- 13) Use the same procedure in problems 6 to 10 except that in these problems the pupils should be directed to write their own arithmetic sentence to describe the action suggested by the problem.

LESSON 2 Concrete Materials Method

There are two types of problems in lesson 2. The action suggested by each type of problem can be represented by arithmetic sentences of the form $N + 3 = 6$ or $3 + N = 6$.

The procedure to follow in this problem solving lesson is as follows:

- 1) Have the class read the problem silently.
- 2) Have one member of the class read the problem aloud.
- 3) Ask the pupils to explain what action is suggested by the problem.
 Example: In problem 1, the action is a combining action. June had 6 leaves. She combined an unknown number of leaves with the 6 she already had. Then she had a total of 24 leaves.

 In problem 3, the action is also a combining action. Jim had an unknown number of cards. He combined 15 more cards with those he already had. Then he had a total of 31 cards.
- 4) Ask the pupils to use their concrete materials to show the action that is suggested by the problem.
 Example: For problem 1, a pupil could place 6 objects together. She could then keep adding more objects to the 6 already in the pile until there are 24 objects in the pile.

 For problem 3, a pupil could place a group of objects in a pile. He could then show the action of combining 15 more objects with the pile he already has. If the number of objects in the pile did not number 31, the pupil could do the same action again until when he combines the 15 objects with the unknown number of objects he has a pile of 31 objects.
- 5) Ask one pupil to demonstrate the action for the class. Encourage discussion of the action being demonstrated.
- 6) Direct the pupils to put an X in front of the arithmetic sentence that best expresses the action suggested by the problem.
- 7) Direct the pupils to compute to find the unknown term. Here the concrete materials could be used to demonstrate that the child must subtract to find the missing term in arithmetic sentences of the form $N + 3 = 6$ or $3 + N = 6$.
- 8) Ask the pupil to write the correct arithmetic sentence and the computation on the blackboard.
- 10) Direct the pupils to complete the verbal sentence that answers the question asked by the problem. Then have one pupil read the sentence.
- 11) Repeat the same procedure for problems 2 to 5.
- 12) Use the same procedure in problems 6 to 10 except that in these problems the pupils should be directed to write their own arithmetic sentence to describe the action suggested by the problem.

LESSON 3

Concrete Materials Method

Lesson 3 contains problems similar to those studied in lessons 1 and 2.

The procedure to follow in this problem solving lesson is as follows:

- 1) Have the class read the first problem silently.
- 2) Have one member of the class read the problem aloud.
- 3) Ask the pupils to explain what action is suggested by the problem. It is important that the pupils be drawn to think about the action that would take place if the situation described by the problem were really taking place. It is the aim of these lessons to make the pupil understand that this suggested action can be represented by an arithmetic sentence.
- 4) Ask the pupils to use their concrete materials to show the action that is suggested by the problem. Allow the pupils enough time to work out their own concrete expression of the action described by the problem. As you move around the room, ask some of the pupils to show you how they would describe the action using their concrete materials. Praise those that make a good effort.
- 5) Ask one pupil to demonstrate the action for the class. Encourage discussion of the action being demonstrated.
- 6) Direct the pupils to write the arithmetic sentence that best expresses the action suggested by the problem.
- 7) Direct the pupils to compute to find the unknown term in the arithmetic sentence. It may be necessary to use discussion to review the computations used in lessons 1 and 2.
- 8) Ask one pupil to write the correct arithmetic sentence and the computation on the blackboard.
- 9) Direct the pupils to verify their own work by comparing it with the one on the blackboard.
- 10) Direct the pupils to complete the verbal sentence that answers the question asked by the problem. Then have one pupil read the sentence that would answer the question in the problem.

LESSON 4 Concrete Materials Method

There are two types of problems in lesson 4. The action suggested by each type of problem can be represented by arithmetic sentences of the form $N - 3 = 6$ or $9 - N = 6$.

The procedure to follow in this problem solving lesson is as follows:

- 1) Have the class read the first problem silently.
- 2) Have one pupil read the problem aloud.
- 3) Ask the pupils to explain what action is suggested by the problem.
Example: In problem 1, the action is a separating action. Tom has a group of pictures. He separates the 8 he gives to his sister from the whole group of pictures. Then he has only 7 pictures left.

In problem 3, the action is also a separating action. Jack has a group of 33 cents. He separates a group of cents from the 33 he had to begin with. Then he has only 10 cents left in the original group.

- 4) Ask the pupils to use their concrete materials to show the action that is suggested by the problem.
Example: For problem 1, a pupil could place a group of objects in a pile. He could then show the action of taking 8 objects away from the group. If the number of objects that remained in the original group was not 7, the pupil could begin again with a different number of objects until when the 8 were separated from the original group, 7 were left over.

For problem 3, the pupil could place 33 objects in a pile. Then he could slide some of the objects away so that only 10 objects remained in the original pile.

- 5) Ask one of the pupils to demonstrate the action for the class.
- 6) Direct the pupils to put an X in front of the arithmetic sentence that best expresses the action suggested by the problem.
- 7) Direct the pupils to compute to find the unknown term in the arithmetic sentence. Through manipulation and discussion, allow the pupils to explore the various possibilities until they decide that they must add to find the missing term for problem 1, and they must subtract to find the missing term for problems like the third problem.
- 8) Ask one pupil to write the arithmetic sentence and the computation on the blackboard.
- 9) Direct the pupils to verify their work by comparing it with the work on the blackboard. Then direct the pupils to complete the verbal sentence and have a pupil read it.

- 10) Repeat the same procedure for problems 2 to 4. Use the same procedure for problems 5 to 10 except that for these problems the pupils should be directed to write their own arithmetic sentences to describe the action suggested by the problem.

LESSONS 5 AND 6 Concrete Materials Method

Lessons 5 and 6 are a review of the types of problems studied in lessons 2 and 4.

The procedure to follow in these problem solving lessons is as follows:

- 1) Direct the pupils to do problems 1 to 5 on their own. Allow sufficient time for them to finish.
- 2) Then beginning with problem 1, direct a pupil to read the problem aloud.
- 3) Lead the pupils in a discussion of the action suggested by the problem.
- 4) Direct the pupils to demonstrate the suggested action with their own materials. Then have one pupil demonstrate the action for the class.
- 5) Ask one pupil to write the arithmetic sentence and the appropriate computation on the blackboard.
- 6) Allow the pupils to discuss the work on the blackboard and direct them to verify their own work.
- 7) Ask a pupil to read the appropriate verbal sentence.
- 8) Use the same procedure for problems 6 to 10.

LESSONS 7 AND 8 Concrete Materials Method

Lessons 7 and 8 are a review of all six types of problems studied in the earlier lessons.

The procedure to use for lessons 7 and 8 is as follows:

- 1) Direct the pupils to do problems 1 to 10 on their own. Allow sufficient time for them to finish.
- 2) Then beginning with problem 1, direct a pupil to read the problem aloud.
- 3) Lead the pupils in a discussion of the action suggested by the problem.
- 4) If the pupils appear to have difficulty expressing the suggested action, direct them to demonstrate the action with their own set of materials. Then have one pupil demonstrate the action for the class.
- 5) Ask one pupil to write the arithmetic sentence and the appropriate computation on the blackboard.
- 6) Allow the pupils to discuss the work on the blackboard and direct them to verify their own work.
- 7) Ask a pupil to read the appropriate verbal sentence.
- 8) Do the same for each of the other problems.

APPENDIX C

ARITHMETIC OUTSIDE THE CLASSROOM

FIRST NAME: _____

LAST NAME: _____

- YES NO Do you receive and handle your own allowance at home?
- YES NO Do you have a bank account of your own in which you make your own deposits?
- YES NO Have you sold anything to make money for yourself?
- YES NO Do you mow lawns or do other work to earn money?
- YES NO Do you play games that require the use of numbers?
- YES NO Do you keep score when you play games?
- YES NO Do you ever use a recipe at home?
- YES NO Do you go to the store alone to buy things?
- YES NO Have you used measurements in making something?
- YES NO Do you have a hobby in which you sometimes use numbers?
- YES NO Did you ever fail a grade at school?

If your answer is yes, which grade did you fail?

one

two

three

APPENDIX D

Item	_____
Item	_____
Item	_____

1. The first item is...

The second item is... The third item is... The fourth item is... The fifth item is... The sixth item is... The seventh item is... The eighth item is... The ninth item is... The tenth item is...

The eleventh item is... The twelfth item is... The thirteenth item is... The fourteenth item is... The fifteenth item is...

The sixteenth item is...

The seventeenth item is... The eighteenth item is... The nineteenth item is... The twentieth item is... The twenty-first item is... The twenty-second item is... The twenty-third item is... The twenty-fourth item is... The twenty-fifth item is...

APPENDIX D

The twenty-sixth item is... The twenty-seventh item is... The twenty-eighth item is... The twenty-ninth item is... The thirtieth item is...

The thirty-first item is... The thirty-second item is... The thirty-third item is... The thirty-fourth item is... The thirty-fifth item is...

The thirty-sixth item is... The thirty-seventh item is... The thirty-eighth item is... The thirty-ninth item is... The fortieth item is...

The forty-first item is... The forty-second item is... The forty-third item is... The forty-fourth item is... The forty-fifth item is... The forty-sixth item is... The forty-seventh item is... The forty-eighth item is... The forty-ninth item is... The fiftieth item is...

The fifty-first item is... The fifty-second item is... The fifty-third item is... The fifty-fourth item is... The fifty-fifth item is...

The fifty-sixth item is... The fifty-seventh item is... The fifty-eighth item is... The fifty-ninth item is... The sixtieth item is...

The sixty-first item is... The sixty-second item is... The sixty-third item is... The sixty-fourth item is... The sixty-fifth item is...

The sixty-sixth item is... The sixty-seventh item is... The sixty-eighth item is... The sixty-ninth item is... The seventieth item is...

The seventy-first item is... The seventy-second item is... The seventy-third item is... The seventy-fourth item is... The seventy-fifth item is...

The seventy-sixth item is... The seventy-seventh item is... The seventy-eighth item is... The seventy-ninth item is... The eightieth item is...

PROBLEM SOLVING TEST

FIRST NAME: _____

LAST NAME: _____

TEACHER: _____

TO THE PUPIL

This is a test of your ability to solve arithmetic problems. Read each problem carefully. Write an arithmetic sentence that tells the action given by the problem. Compute to find the answer for the problem. Then write the number on the blank in the sentence that answers the problem.

Study SAMPLE 1 below and be sure you know what to do.

Then do SAMPLE 2.

SAMPLE 1

Jane had 10 pennies. She spent 4 of her pennies for candy. Then Jane had how many pennies?

Write an arithmetic sentence

Computation

$$\underline{10 - 4 = N}$$

$$\begin{array}{r} 10 \\ - 4 \\ \hline 6 \end{array}$$

Then Jane had 6 pennies.

SAMPLE 2

Joey had 19 postcards. Then his uncle sent him 6 more. How many postcards did Joey have after his uncle sent him 6 more?

Write an arithmetic sentence

Computation

Joey had _____ postcards after his uncle sent him 6 more.

DO NOT START THE TEST UNTIL YOUR

TEACHER TELLS YOU TO DO SO.

- 1) The children at school had 126 new books at the beginning of the year. During the year some of the books were lost. At the end of the year, only 79 books were left. How many books had been lost?

Write the arithmetic sentence

Computation

_____ books had been lost during the year.

- 2) Ricky's brother sent him 37 stamps. Then Ricky had 102 stamps in his collection. How many stamps did Ricky have before his brother sent him some more?

Write the arithmetic sentence

Computation

Ricky had _____ stamps in his collection before his brother sent him some more.

- 3) 423 children were in school. At recess, 285 went out to play. How many children stayed in the school?

Write the arithmetic sentence

Computation

_____ children stayed in the school.

- 4) Tom had 409 stamps in his collection. His uncle gave him 175 more. How many are in Tom's collection now?

Write the arithmetic sentence

Computation

Now there are _____ stamps in Tom's collection.

- 5) Dick had \$2.95 to spend. He bought a toy that cost \$1.58. How much money did Dick have left after he bought the toy?

Write the arithmetic sentence

Computation

Dick had \$_____ left after he bought the toy.

- 6) There are 76 boys and 45 girls in our school. How many children are in our school?

Write the arithmetic sentence

Computation

There are _____ children in our school.

- 7) Joey had \$2.64. Then his father gave him some more money. When he counted his money, he now had \$2.95. How much money did Joey's father give to Joey?

Write the arithmetic sentence

Computation

Joey's father gave Joey \$_____.

- 8) Carol planted 14 flowers before school. After school Carol planted some more flowers. Then Carol had planted 32 flowers in all. How many flowers did Carol plant after school?

Write the arithmetic sentence

Computation

Carol planted _____ flowers after school.

- 9) Don had some bottle caps in a box. On the way to school Don gave 42 to Jack. Then Don had 261 bottle caps left. How many bottle caps did Don have before he gave Jack some?

Write the arithmetic sentence

Computation

Don had _____ bottle caps before he gave Jack some.

- 10) Ann made 26 cupcakes. Then her sister made some more. All together they had 66 cupcakes. How many cupcakes did Ann's sister make?

Write the arithmetic sentence

Computation

Ann's sister made _____ cupcakes.

- 11) Jane bought a box of 24 candies. Then she had 53 candies. How many candies did Jane have to begin with?

Write the arithmetic sentence

Computation

Jane had _____ candies before she bought some more.

- 12) John had some marbles. He gave 17 to his sister. Then John had 34 marbles left. How many marbles did John have before he gave some to his sister?

Write the arithmetic sentence

Computation

John had _____ marbles before he gave some to his sister.

- 13) A farmer bought 175 chickens. Then he had 425 chickens. How many chickens did the farmer have before he bought some more?

Write the arithmetic sentence

Computation

The farmer had _____ chickens before he bought some more.

- 14) Farmer Brown had 45 pigs. He sold 24 pigs. How many pigs did he have left?

Write the arithmetic sentence

Computation

Farmer Brown had _____ pigs left.

- 15) Mary read 27 pages in her book on Monday. She read some more on Tuesday. Then she had read 61 pages in all. How many pages did Mary read on Tuesday?

Write the arithmetic sentence

Computation

Mary read _____ pages on Tuesday.

- 16) There were 62 children on the bus. 34 of them got off at the swimming pool. How many children were left on the bus?

Write the arithmetic sentence

Computation

There were _____ children left on the bus.

- 17) The book John was reading had 238 pages in it. After school John read until there were only 150 pages left. How many pages did John read after school?

Write the arithmetic sentence

Computation

John read _____ pages after school.

- 18) Carol's mother put 60 candies in a dish. After school Carol's friends ate some of the candies. Then there were only 23 left. How many candies did Carol's friends eat?

Write the arithmetic sentence

Computation

Carol's friends ate _____ of the candies.

- 19) Last year there were 386 children in our school. This year there are 213 more children. How many children go to our school this year?

Write the arithmetic sentence

Computation

_____ children go to our school this year.

- 20) After John was given \$1.65, he had \$4.73 in all. How much money did John have before he was given the \$1.65?

Write the arithmetic sentence

Computation

John had \$ _____ before he was given the \$1.65.

- 21) Pat bought a record for \$1.89. Then he had \$3.11 left. How much money did Pat have before he bought the record?

Write the arithmetic sentence

Computation

Pat had \$ _____ before he bought the record.

- 22) Grade 3 sold 207 tickets for a puppet show. Grade 4 sold 235 tickets. What was the total number of tickets sold?

Write the arithmetic sentence

Computation

The total number of tickets sold was _____.

- 23) Bob had \$3.50 to buy a game. After he bought the game he had \$1.41 left. How much did Bob pay for the game?

Write the arithmetic sentence

Computation

Bob paid \$ _____ for the game.

- 24) A farmer sold 56 turkeys. Then he still had 109 turkeys. How many turkeys did he have before he sold some?

Write the arithmetic sentence

Computation

The farmer had _____ turkeys before he sold some.

DIRECTIONS FOR ADMINISTERING THE PROBLEM SOLVING TEST (PRETEST)

- 1) Direct the pupils to take out a sharp pencil and an eraser.
- 2) Ask the pupils to fill in the blanks for their names and their teacher's name.
- 3) READ: "TO THE PUPIL"

This is a test of your ability to solve arithmetic problems. Read each problem carefully. Write an arithmetic sentence that tells the action given by the problem. Compute to find the answer for the problem. Then write the number on the blank in the sentence that answers the problem.

- 4) Say:

Now look at sample 1 and read it to yourself silently while I read it aloud. It says:

"Sample 1

Jane had 10 pennies. She spent 4 of her pennies for candy. Then Jane had how many pennies?

Think about the action that the problem suggests. What arithmetic sentence tells what Jane did?

Encourage discussion:

Yes, the arithmetic sentence $10 - 4 = N$ means that Jane had 10 pennies. She spent 4 of them. Then she had N pennies left.

How do you find the number to replace N?

Discussion:

We subtract 10 minus 4 equals 6.

Then what number do we write on the blank to complete the answer?

Discussion:

The number 6. Our answer is then "Jane had 6 pennies."

Now try sample 2 in the same way.

Allow the pupils some time to try sample 2 on their own. Then SAY:

Read sample 2 again as I read it aloud. It says:

Sample 2: Joey had 19 postcards. Then his uncle sent him 6 more.

How many postcards did Joey have after his uncle sent him 6 more?

What arithmetic sentence best tells what action is suggested by the problem?

Encourage discussion:

The arithmetic sentence that best tells the action suggested by sample 2 is $19 + 6 = s$. The 19 shows how many postcards Joey had to begin with. The +6 shows that Joey was given 6 more postcards. The s stands for the unknown number of postcards Joey had altogether after his uncle sent him some more.

Now, how do you find the number that should replace s in the arithmetic sentence?

Discussion:

Yes, you would add 19 plus 6. The sum is 25. Then the sentence would say, "Joey had 25 more postcards after his uncle sent him 6 more."

After you are sure that all the pupils understand how to do the test, say:

There are 24 problems in this test. Try to do each of the problems in the same way as you did sample 1 and 2. You will have 30 minutes to do the 24 problems. Try not to spend too much time on any of the problems. If you cannot do one of the problems, leave it and go on to the next problem. Then when you finish the test you can go back to do the ones you missed.

When you are sure the pupils understand, say: START THE TEST. Mark the starting time on the blackboard. Please do not help the pupils but move around the room to make certain that each pupil goes on to the next page each time he completes a page.

After 30 minutes say STOP! Then collect the papers.

Starting time _____ 30 minutes Stopping time _____

DIRECTIONS FOR ADMINISTERING THE PROBLEM SOLVING TEST (POST TEST)

- 1) Ask the pupils to fill in the blanks for their names and their teacher's name.
- 2) READ:
This is the same problem solving test that you did two weeks ago. Last time you did this test, many of you made mistakes. Since your last test, you have done 8 lessons to improve your ability to solve arithmetic problems. Try your best on this test so it will show how much you have learned from the lessons you did.
- 3) READ: "TO THE PUPIL" (on the front of the test)
- 4) SAY:
Now look at sample 1 and read it to yourself silently while I read it aloud. It says:

Sample 1

Jane had 10 pennies. She spent 4 of her pennies for candy.
Then Jane had how many pennies?

Think about the action that the problem suggests. What arithmetic sentence tells what Jane did?

Encourage discussion:

Yes, the arithmetic sentence $10 - 4 = N$ means that Jane had 10 pennies. She spent 4 of them. Then she had N pennies left.

How do you find the number to replace N?

Discussion:

We subtract 10 minus 4 equals 6.

Then what number do we write on the blank to complete the answer?

Discussion:

The number 6. Our answer is "Then Jane had 6 pennies."
Now try sample 2 in the same way.

Allow the pupils some time to try sample 2 on their own. Then say:

Read sample 2 again as I read it aloud. It says:

Sample 2: Joey had 19 postcards. Then his uncle sent him 6 more.
How many postcards did Joey have after his uncle sent him 6 more?
What arithmetic sentence best tells what action is suggested by the problem?

Encourage discussion:

The arithmetic sentence that best tells the action suggested by sample 2 is $19 + 6 = s$. The 19 shows how many postcards Joey had to begin with. The + 6 shows that Joey was given 6 more postcards. The s stands for the unknown number of postcards Joey had altogether after his uncle sent him some more.

Now, how do you find the number that should replace s in the arithmetic sentence.

Discussion:

Yes, you would add 19 plus 6. The sum is 25. Then the sentence would say, "Joey had 25 postcards after his uncle sent him 6 more."

After you are sure that all the pupils understand how to do the test, say:

There are 24 problems in this test. Try to do each of the problems in the same way as you did sample 1 and 2. You will have 30 minutes to do the 24 problems. Try not to spend too much time on any of the problems. If you cannot do one of the problems, leave it and go on to the next problem. Then when you finish the test you can go back to do the ones you missed.

When you are sure the pupils understand, say: START THE TEST. Mark the starting time on the blackboard. Please do not help the pupils but move around the room to make certain that each pupil goes on to the next page each time he completes a page.

After 30 minutes say STOP! Then collect the papers.

Starting time _____ 30 minutes Stopping time _____

APPENDIX E

ARITHMETIC COMPUTATION

FIRST NAME: _____

LAST NAME: _____

TEACHER: _____

Work each example carefully and write your answer in the proper place.

The sample below is marked correctly.

Sample

$$\begin{array}{r} 5 \\ +40 \\ \hline 45 \end{array}$$

$$\begin{array}{r} 7 \\ +7 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ +90 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \\ +33 \\ \hline \end{array}$$

$$\begin{array}{r} 32 \\ +87 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ +3 \\ \hline \end{array}$$

$$\begin{array}{r} 86 \\ -43 \\ \hline \end{array}$$

$$\begin{array}{r} 58 \\ -30 \\ \hline \end{array}$$

$$\begin{array}{r} 524 \\ -103 \\ \hline \end{array}$$

$$\begin{array}{r} 36 \\ 42 \\ +23 \\ \hline \end{array}$$

$$\begin{array}{r} 412 \\ 354 \\ +265 \\ \hline \end{array}$$

$$\begin{array}{r} 57 \\ 106 \\ +679 \\ \hline \end{array}$$

$$\begin{array}{r} 275 \\ 116 \\ +9 \\ \hline \end{array}$$

$$\begin{array}{r} 549 \\ -401 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ -37 \\ \hline \end{array}$$

$$\begin{array}{r} 524 \\ -376 \\ \hline \end{array}$$

$$\begin{array}{r} 860 \\ -347 \\ \hline \end{array}$$

$$\begin{array}{r} 4280 \\ 1566 \\ +6143 \\ \hline \end{array}$$

$$\begin{array}{r} 1728 \\ 8600 \\ +6493 \\ \hline \end{array}$$

$$\begin{array}{r} 1402 \\ 1042 \\ +4210 \\ \hline \end{array}$$

$$\begin{array}{r} 37 \\ 273 \\ +789 \\ \hline \end{array}$$

$$\begin{array}{r} 801 \\ -667 \\ \hline \end{array}$$

$$\begin{array}{r} 6579 \\ -2868 \\ \hline \end{array}$$

$$\begin{array}{r} 2631 \\ -1742 \\ \hline \end{array}$$

$$\begin{array}{r} 9753 \\ -2468 \\ \hline \end{array}$$

DIRECTIONS FOR ADMINISTERING THE COMPUTATION TEST

- 1) Have each pupil take out a sharp pencil and an eraser.
- 2) Distribute the papers and have the pupils place them on their desks face down.
- 3) When the pupils are all prepared have them turn over their papers and fill in the blanks for their names and the name of their teacher.

- 4) Say:

This is a test of your ability to add and to subtract.
The directions at the top of the test say, "Work each example carefully and write your answer in the proper place. The sample below is marked correctly."

- 5) Discuss the sample with the pupils.

- 6) When you are sure they all understand, say:

You will have 15 minutes to do the 24 questions in this test. Be sure to watch the signs that tell you to add or subtract.

- 7) When they are all set, say: START NOW!

- 8) Let the pupils work on the test independently for 15 minutes then say STOP!

- 9) Collect the papers.

Starting time _____ 15 minutes Stopping time _____

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